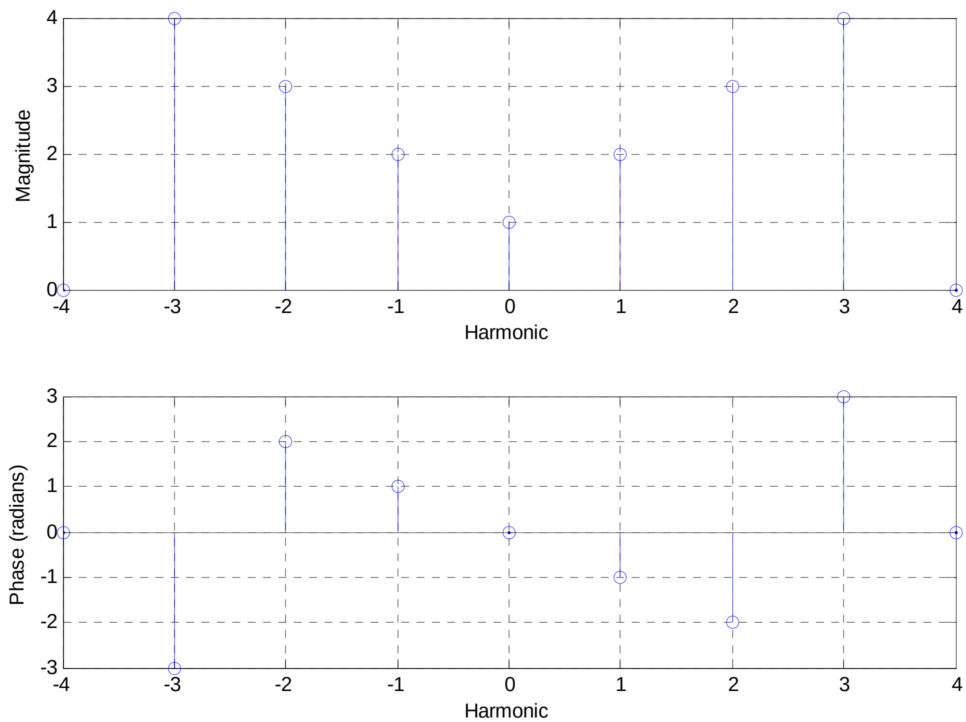
ECE 300
Signals and Systems
Homework 6

<u>Due Date:</u> Tuesday October 16, 2007 at the beginning of class

<u>Exam 2, Thursday October 18, 2007</u>

<u>Problems:</u>

1. Assume $x(t)$ has the spectrum shown below (the phase is shown in radians) and a fundamental frequency $\omega_o = 2 \text{ rad/sec}$:



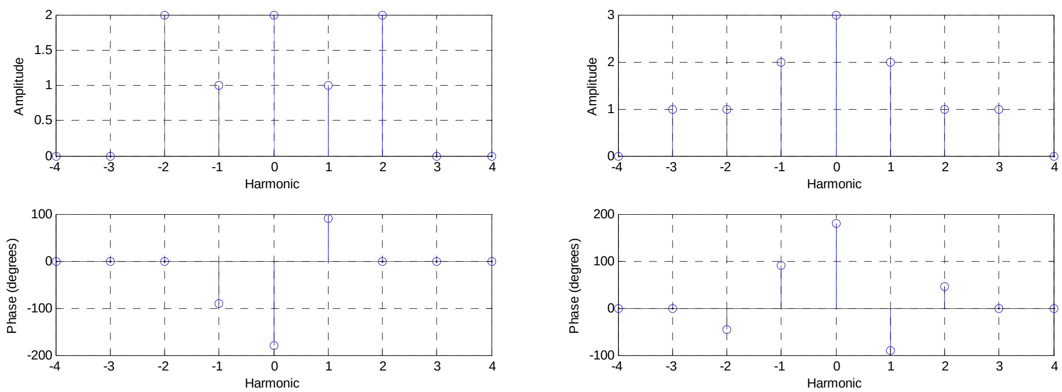
Assume $x(t)$ is the input to a system with the transfer function

$$H(\omega) = \begin{cases} e^{-j\omega} & 1 \le |\omega| < 3 \\ 2e^{-j2\omega} & 3 < |\omega| < 5 \\ 0 & else \end{cases}$$

Determine an expression for the steady state output $y(t)$. Be as specific as possible, simplifying all values and using actual numbers wherever possible.

2. ZTM, Problem 3-16.

3. The output of a LTI system, $y(t)$, has the following spectrum shown on the left, while the system transfer function, $H(k\omega_o)$, has the spectrum shown on the right. Assume all angles are multiples of 45 degrees.



a) Determine (sketch) the spectrum (magnitude and phase) of the input to the system, $x(t)$.

b) If $x(t)$ has the fundamental period $T = 2$ seconds, determine an analytical expression for $x(t)$ in terms of sine, cosines, and constants.

4. Assume two periodic signals have the Fourier series representations

$$x(t) = \sum X_k e^{jk\omega_o t} \qquad y(t) = \sum Y_k e^{jk\omega_o t}$$

For the following system (input/output) relationships:

a) $y(t) = bx(t-a)$
b) $y(t) = b\dot{x}(t-a)$
c) $y(t) = bx(t)\cos(\omega_o t)$ (Answer: $Y_n = \frac{b}{2}\left(X_{n-1} + X_{n+1}\right)$)
d) $\ddot{y}(t) + \frac{2\varsigma}{\omega_n}\dot{y}(t) + \frac{1}{\omega_n^2}y(t) = Kx(t)$

i) write Y_k in terms of the X_k
ii) If possible, determine the system transfer function $H(j\omega)$
iii) A system must be both linear and time-invariant to have a transfer function. If you cannot determine the transfer function, indicate which system property is not satisfied (**L** or **TI).**

5. Assume $x(t)$ has the Fourier series representation $x(t) = \sum X_k e^{jk\omega_o t}$ and fundamental period T_o. The function $y(t)$ is related to $x(t)$ through the relationship $y(t) = x\left(\frac{t}{b}\right)$.

a) Determine the period of $y(t)$ in terms of T_o (the period of $x(t)$) and fundamental frequency for $y(t)$ in terms of ω_o (the fundamental frequency for $x(t)$)

b) Set up the integral to determine the Fourier series coefficients Y_k in terms of the parameters determined in part a (the integral should be centered at 0), and determine how Y_k is related to X_k

c) Starting from the relationship $x(t) = \sum X_k e^{jk\omega_o t}$ and making a simple substitution, show how we can determine the results from part b.

This problem demonstrates that compression or expansion of a signal does not change the Fourier series coefficients, it only changes the fundamental frequency.