

ECE 300
Signals and Systems
Homework 7

Due Date: Friday October 21 at 1 PM **Exam 2, Tuesday October 18**

Reading: K & H, pp. 161-192

Problems

1. By evaluating the integral by hand, show that the Fourier transform of $x(t) = e^{-t}u(t)$ is given by

$$X(\omega) = \frac{1}{1+j\omega} = \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}(j\omega) \left[\frac{180}{\pi} \right] \text{ degrees}$$

2. By evaluating the integral by hand, show that the Fourier transform of $x(t) = e^{-|t|}$ is

$$X(\omega) = \frac{2}{1+\omega^2}$$

3) In this problem you will utilize the Matlab program **Fourier_Series.m** on the class website (download by **right clicking, select save target as**, and **saving as a text document**). The arguments to this function are the initial and final times of a single period (the period starts at **Tlow** and ends at **Thigh**) and **N**, the number of terms to use (in addition to the average value term).

- a. Write a function that takes as arguments c_0 , c , and T (or T_0 , the fundamental frequency) and plots the amplitude $T|c_k|$ versus $k\omega_0$ and the phase $\angle c_k$ (in degrees) versus $k\omega_0$. You should use the subplot command and plot both on one page. You should use the command **orient tall** before any plotting to use more of the page. Some Matlab commands you might find useful are **angle**, **length**, and **abs**. Instead of using the stem command, you should use the plot command and plot discrete points, like dots ('.').
- b. By using the axis command, limit the axes to the range -8 to 8 and from 0 to the maximum value of $T|c_k|$. The **max** command may prove useful here.
- c. Change the function **fcn** so we are plotting the spectrum of $x(t) = e^{-t}u(t)$. You should be sure to look at the Fourier series representation to verify everything is correct.

- d. Add plots of the magnitude and phase of $X(\omega)$ on the existing plots. Use a solid line type and be sure to add legends. If you have done everything correctly, and you type `Fourier_Series(-4,4,100)`, you should get the plot shown in Figure 1. Be sure to modify the title and axes so they look like those in this figure (to get subscripts type c_k , to get T0 type T_0 , and to get ω_0 type ω_0)

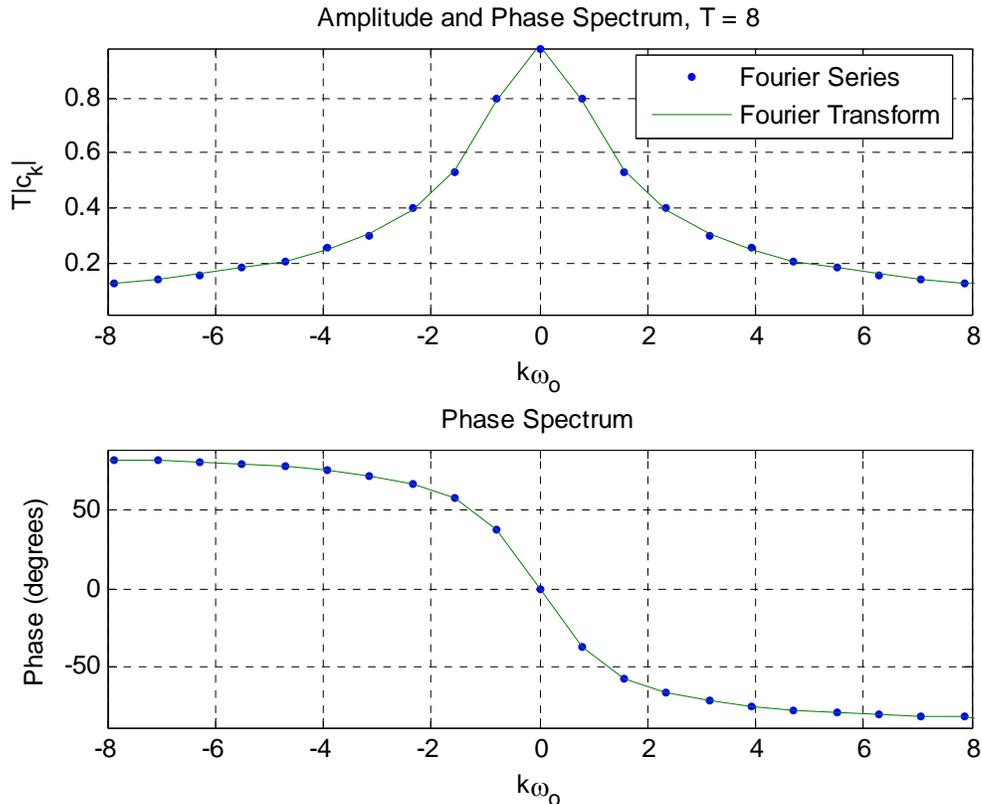


Figure 1: Example plots for part d.

- e. Change **[Tlow, Thigh]** from $[-8,8]$, $[-16, 16]$ and $[-32,32]$ and turn in your plots (these are just two of the three inputs to your function). Keep the number of points at $N = 100$. Do not change $x(t)$. Here we are increasing the period of the function $x(t)$ to demonstrate that the Fourier transform is just a Fourier series in the limit as $T \rightarrow \infty$, $k\left(\frac{2\pi}{T}\right) = k\omega_0 \rightarrow \omega$, and $Tc_k \rightarrow X(\omega)$

- f) Redo the above for $x(t) = e^{-|t|}$. Turn in 4 plots for this part.