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ECE-205

Exam 3

Winter 2013

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/15

Problem 2 ________/10

Problem 3 ________/20

Problem 4 ________/15

Problems 5 ________/20

Problems 6 ________/20

Total ____________

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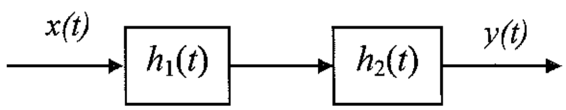
1) (15 Points) Consider the LTI systems with the following impulses responses:

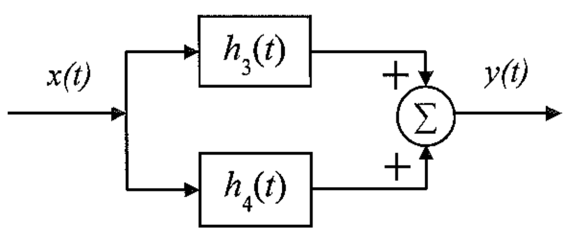
$h_1(t)=\delta(t-2)$, $h_2(t)=\delta(t-1)-3\delta(t-3)$, $h_3(t)=e^{(t+1)}\delta(t+1)$, $h_4(t)=u(t-2)-\delta(t+1)$

i) Fill in the following table. You do not need to show any work.

	Causal? (Y/N)	BIBO Stable? (Y/N)
$h_1(t)=\delta(t-2)$	Y	Y
$h_2(t)=\delta(t-1)-3\delta(t-3)$	Y	Y
$h_3(t)=e^{(t+1)}\delta(t+1)$	N	Y
$h_4(t)=u(t-2)-\delta(t+1)$	N	N

ii) Determine the overall impulse response (the impulse response between input $x(t)$ and output $y(t)$) of the following interconnected systems. You must show your work for a full credit.





$$h_1(t)*h_2(t) = \int_{-\infty}^{\infty} \delta(\lambda-2)\left[\delta(t-\lambda-1)-3\delta(t-\lambda-3)\right]d\lambda$$

$$= \int_{-\infty}^{\infty} \delta(\lambda-2)\delta(t-\lambda-1)\,d\lambda - 3\int_{-\infty}^{\infty}\delta(\lambda-2)\delta(t-\lambda-3)\,d\lambda$$

$$= \delta(t-1-2) - 3\delta(t-3-2) = \boxed{\delta(t-3)-3\delta(t-5)}$$

$$h_3(t)+h_4(t) = e^{(t+1)}\delta(t+1) + u(t-2) - \delta(t+1)$$

$$= e^{(-1+1)}\delta(t+1) + u(t-2) - \delta(t+1)$$

$$= \delta(t+1) + u(t-2) - \delta(t+1)$$

$$= \boxed{u(t-2)}$$

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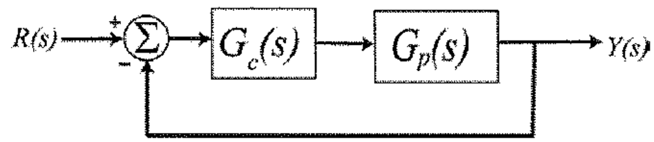
2) (10 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{2}{s+3}$



a) Determine the settling time of the plant alone (assuming there is no feedback)

(Dominant) pole at $s = -3$

$\Rightarrow \boxed{T_s = 4/3}$

b) Determine the steady state error for plant alone assuming the input is a unit step (simplify your answer as much as possible)

unit step $\Rightarrow A = 1$

$e_{ss} = A[1 - G_p(0)] = 1 - G_p(0) = 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$

c) For a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function $G_0(s)$

$$G_0(s) = \frac{\frac{2k_p}{s+3}}{1 + \frac{2k_p}{s+3}} = \boxed{\frac{2k_p}{s+3+2k_p}}$$

d) Determine the settling time of the closed loop system, in terms of k_p

(Dominant) pole at $s = -(3+2k_p)$

$$\Rightarrow \boxed{T_s = \frac{4}{3+2k_p}}$$

e) Determine the steady state error of the closed loop system for a unit step, in terms of k_p (simplify your answer as much as possible)

unit step input $\Rightarrow A = 1$

$$e_{ss} = A[1 - G_0(0)] = 1 - G_0(0) = 1 - \frac{2k_p}{3+2k_p}$$

$$= \boxed{\frac{3}{3+2k_p}}$$

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3) (20 points) Determine

a) the *impulse response* of $H(s)=\dfrac{4}{s^2+2s+5}$

b) the *unit step response* of $H(s)=\dfrac{e^{-2s}}{(s+1)^2}$

a) $$H(s)=\underbrace{\frac{4}{s^2+2s+5}=\frac{4}{(s+1)^2+(2)^2}}_{\text{completing the square}}=\left(\frac{2}{(s+1)^2+(2)^2}\right)\cdot 2$$

by $$\mathcal{L}\{e^{-\alpha t}\sin(\omega_0 t)u(t)\}=\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$$

$$\Rightarrow \boxed{h(t)=2e^{-t}\sin(2t)u(t)}$$

b) Unit step input $x(t)=u(t) \longleftrightarrow X(s)=1/s$

unit step response $Y(s)=H(s)X(s)=\frac{1}{s}H(s)=\dfrac{e^{-2s}}{s(s+1)^2}$

Let $G(s)=\dfrac{1}{s(s+1)^2}$

By partial fractions:

$$G(s)=\frac{1}{s(s+1)^2}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{(s+1)^2}$$

- By cover-up method: $A=1$, $C=-1$
- multiply by s and taking $s\to\infty$: $0=A+B \Rightarrow B=-1$

$$\Rightarrow G(s)=\frac{1}{s}-\frac{1}{s+1}-\frac{1}{(s+1)^2} \longleftrightarrow g(t)=u(t)-e^{-t}u(t)-te^{-t}u(t)$$

Finally, $Y(s)=e^{-2s}G(s) \longleftrightarrow y(t)=g(t-2)$

$$\therefore \boxed{y(t)=\left[1-e^{-(t-2)}-(t-2)e^{-(t-2)}\right]u(t-2)}$$

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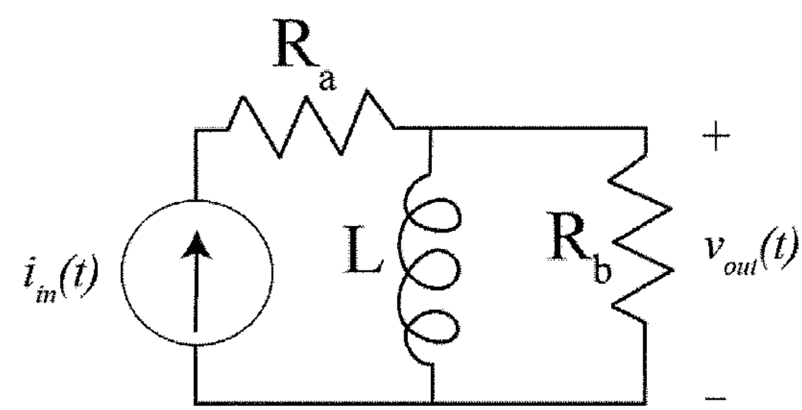
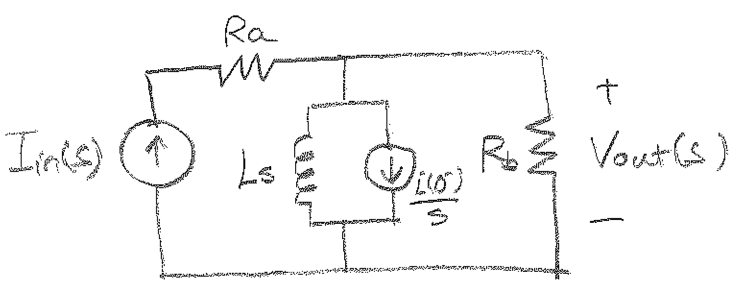
4) (15 points) For the following circuit



- Determine the ZIR
- Determine the ZSR
- Determine the transfer function



$$I_n(s) = \frac{V_{out}(s)}{Ls} + \frac{i(0^-)}{s} + \frac{V_{out}(s)}{R_b}$$

$$\Rightarrow V_{out}(s) = +\frac{R_b L s}{Ls + R_b} I_{in}(s) - \frac{R_b L}{Ls + R_b} i(0^-)$$

$$= \underbrace{+\frac{R_b \cdot s}{s + R_b/L} I_{in}(s)}_{ZSR} \underbrace{- \frac{R_b}{s + R_b/L} i(0^-)}_{ZIR}$$

$\therefore$
$$\boxed{\begin{aligned} ZIR &= -\frac{R_b}{s + R_b/L} i(0^-) \\ ZSR &= +\frac{R_b \cdot s}{s + R_b/L} I_{in}(s) \end{aligned}}$$

Transfer function:
$$\boxed{H(s) = \frac{V_{out}(s)}{I_n(s)} = +\frac{R_b \cdot s}{s + R_b/L}}$$

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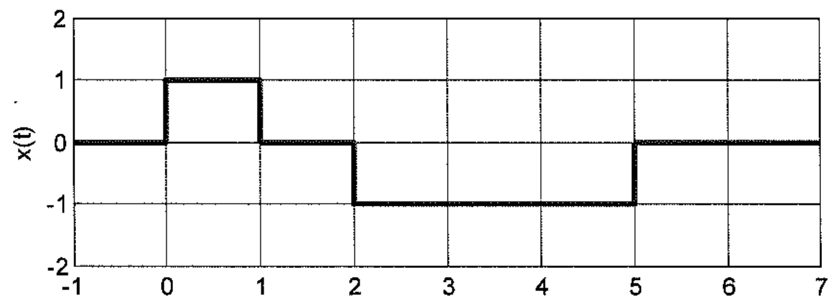
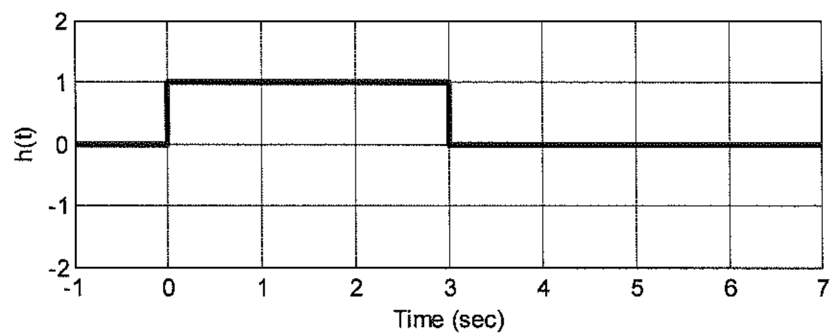
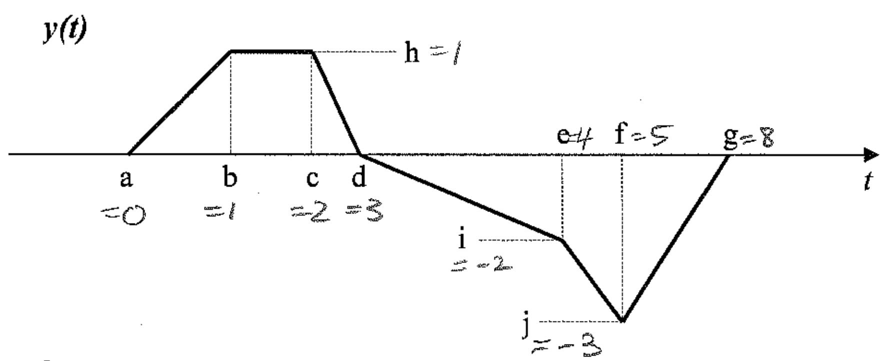
5) (20 Points) An LTI system has input, impulse response, and output as shown below. Determine numerical values for the parameters *a-j*. Note that parameters *a-g* correspond to *times*, and *h-j* correspond to *amplitudes*.

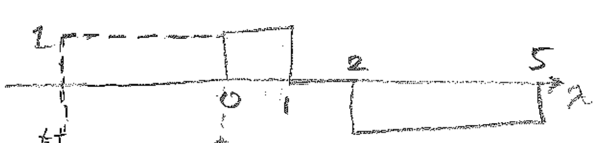
Note that the output graph is only an approximate sketch of the output. Do not try to read values from this sketch.



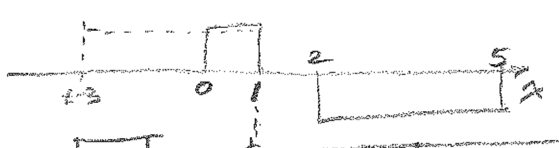




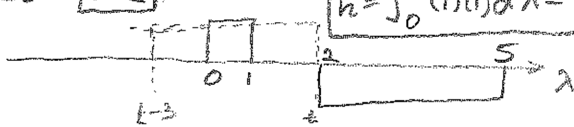
a) $t=0 \Rightarrow \boxed{a=0}$



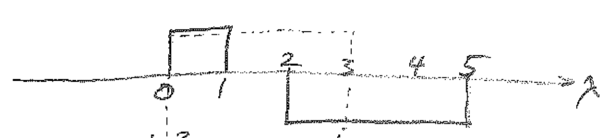
b) $t=2 \Rightarrow \boxed{b=1}$



c) $t=3 \Rightarrow \boxed{c=2}$ $\boxed{h=\int_0^1 (1)(1)\,d\lambda = 1}$

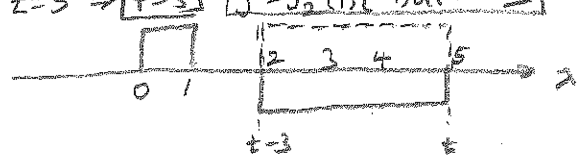


d) $t=3 \Rightarrow \boxed{d=3}$



e) $t=4 \Rightarrow \boxed{e=4}$ $\boxed{i=\int_2^4 (1)(-1)\,dt = -2}$

f) $t=5 \Rightarrow \boxed{f=5}$ $\boxed{j=\int_2^5 (1)(-1)\,dt = -3}$



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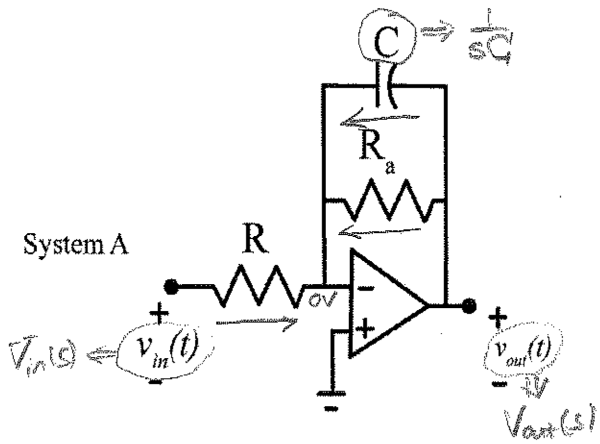
6) (20 points) The following figure shows three different circuits, which are subsystems for a larger system. We can write the transfer functions for these systems as

$$G_a(s) = \frac{-K_{low}\omega_{low}}{s+\omega_{low}} \qquad G_b(s) = \frac{-K_{high}s}{s+\omega_{high}} \qquad G_c(s) = -K_{ap}$$

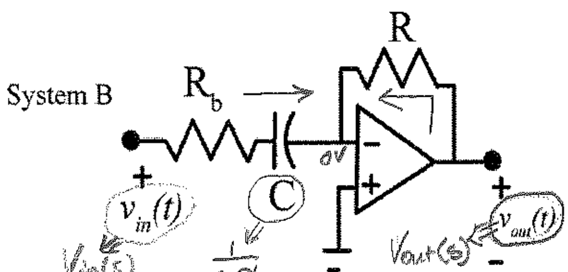
Determine the parameters $K_{low}, \omega_{low}, K_{high}, \omega_{high}$, and K_{ap} in terms of the parameters given (the resistors and capacitors).



$$\frac{V_{in}(s)}{R} + \frac{V_{out}(s)}{R_a} + \frac{V_{out}(s)}{1/(sC)} = 0$$

$$\Rightarrow \frac{V_{out}(s)}{V_{in}(s)} = \frac{-1/(RC)}{s + 1/(R_aC)}$$

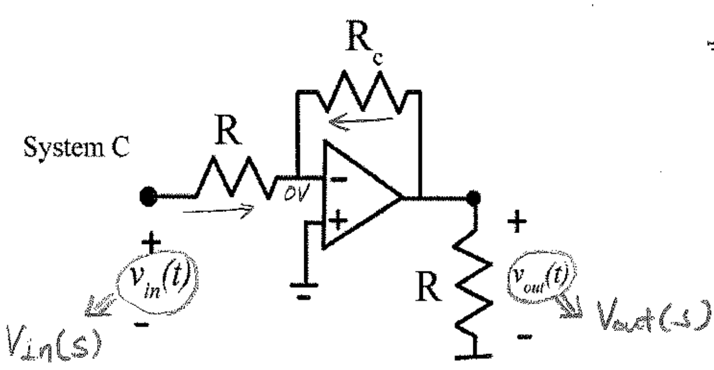
$$\Rightarrow \boxed{K_{low} = R_a/R}, \boxed{\omega_{low} = 1/(R_aC)}$$



$$\frac{V_{in}(s)}{R_b + \frac{1}{sC}} + \frac{V_{out}(s)}{R} = 0$$

$$\Rightarrow \frac{V_{out}(s)}{V_{in}(s)} = -\frac{RCs}{R_bCs+1} = -\frac{(R/R_b)s}{s + 1/(R_bC)}$$

$$\Rightarrow \boxed{\begin{array}{l} K_{high} = R/R_b \\ \omega_{high} = \frac{1}{R_bC} \end{array}}$$



$$\frac{V_{in}(s)}{R} + \frac{V_{out}(s)}{R_c} = 0$$

$$\Rightarrow \frac{V_{out}(s)}{V_{in}(s)} = -\frac{R_c}{R}$$

$$\Rightarrow \boxed{K_{ap} = \frac{R_c}{R}}$$

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