Name Solutions CM ____________

ECE-205

Exam 3

Winter 2012

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/15

Problem 2 ________/15

Problem 3 ________/20

Problem 4 ________/20

Problem 5 ________/15

Problem 6 ________/15

Total ____________

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1) (15 points) For the following transfer functions, determine the *unit step response* of the system. *Do not forget any necessary unit step functions.*

a) $H(s) = \frac{e^{-3s}}{(s+1)^2}$

b) $H(s) = \frac{1}{s^2 + 4s + 8}$

a) $Y(s) = H(s)\frac{1}{s} = \frac{e^{-3s}}{s(s+1)^2} = e^{-3s}G(s)$

$G(s) = \frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$ $\quad A = 1 \quad C = -1$

$\times s$, let $s \to \infty$ $\quad 0 = A + B \quad B = -A = -1$

$g(t) = (1 - e^{-t} - te^{-t})u(t)$

$y(t) = g(t-3) = \boxed{\left[1 - e^{-(t-3)} - (t-3)e^{-(t-3)}\right]u(t-3) = y(t)}$

b) $Y(s) = H(s)\frac{1}{s} = \frac{1}{s[(s+2)^2+4]} = \frac{A}{s} + \frac{B\,2}{(s+2)^2+4} + C\frac{(s+2)}{(s+2)^2+4}$

$A = \frac{1}{8}$ $\quad \times s$, let $s \to \infty$ $\quad 0 = A + C \quad C = -1/8$

let $s = -2$ $\quad \frac{1}{-8} = \frac{-1}{16} + \frac{B}{2} \quad -2 = -1 + 8B \quad B = -1/8$

$\boxed{y(t) = \frac{1}{8}\left[1 - e^{-2t}\sin(2t) - e^{-2t}\cos(2t)\right]u(t)}$

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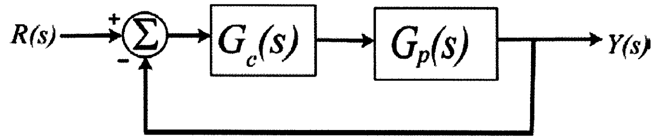
2) (15 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{3}{s+5}$



a) Determine the settling time of the plant alone (assuming there is no feedback) $\boxed{T_s = \frac{4}{5}}$

b) Determine the steady state error for plant alone assuming the input is a unit step (simplify your answer)

$$e_{ss} = 1 - \tfrac{3}{5} = \boxed{\frac{2}{5} = e_{ss}}$$

c) For a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function $G_0(s)$

$$\boxed{G_0(s) = \frac{3K_p}{s+5+3K_p}}$$

d) Determine the settling time of the closed loop system , in terms of k_p $\boxed{T_s = \frac{4}{5+3K_p}}$

e) Determine the steady state error of the closed loop system for a unit step, in terms of k_p (simplify your answer)

$$e_{ss} = 1 - \frac{3K_p}{5+3K_p} = \boxed{\frac{5}{5+3K_p} = e_{ss}}$$

f) For and integral controller, $G_c(s) = \dfrac{k_i}{s}$, determine the closed loop transfer function $G_0(s)$ and the steady state error for a unit step in terms of k_i

$$\boxed{G_0(s) = \frac{3K_i}{s(s+5)+3K_i}} \qquad \boxed{e_{ss} = 0}$$

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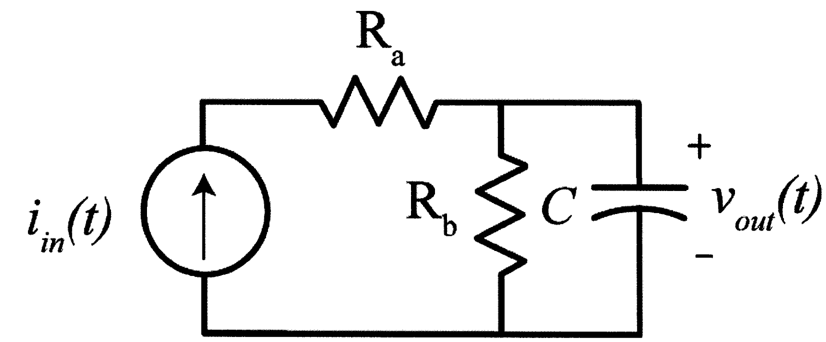
3) (20 points) For the following circuit,



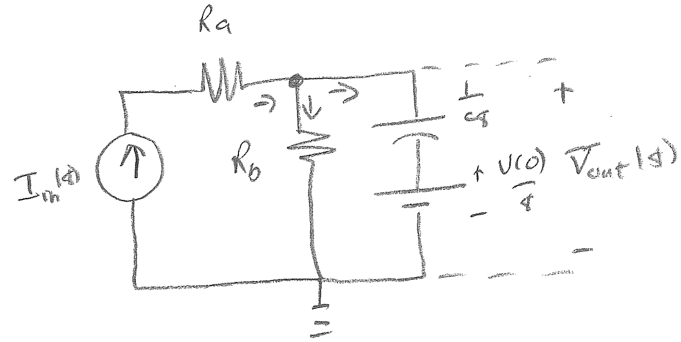
Determine expressions for the following in terms of the parameters given

a) the zero input response (ZIR)

b) the zero state response (ZSR)

c) the transfer function

d) the impulse response



$$I_{in}(s) = \frac{V_{out}(s)}{R_b} + \frac{V_{out}(s) - \frac{V(0)}{s}}{\frac{1}{Cs}} = -CV(0) + V_{out}(s)\left[\frac{1}{R_b} + Cs\right]$$

$$I_{in}(s) + CV(0) = V_{out}(s)\left[\frac{R_bCs+1}{R_b}\right]$$

$$V_{out}(s) = \underbrace{\left[\frac{R_b \quad I_{in}(s)}{R_bCs+1}\right]}_{ZSR} + \underbrace{\left[\frac{R_bCV(0)}{R_bCs+1}\right]}_{ZIR}$$

$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \boxed{\frac{R_b}{R_bCs+1} = H(s)} = \frac{R_b}{R_bC(s+1/R_bC)} = \frac{1}{C}\frac{1}{s+1/R_bC}$$

$$\boxed{h(t) = \frac{1}{C}e^{-t/R_bC}u(t)}$$

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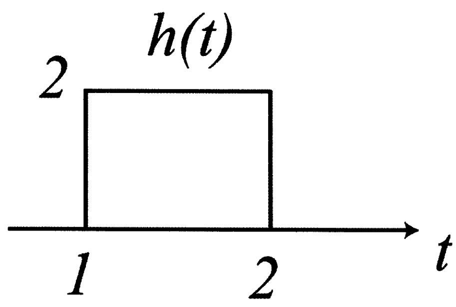
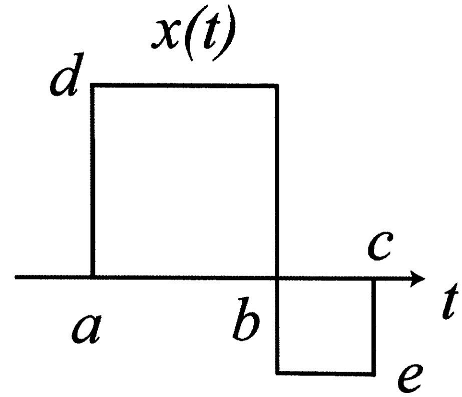
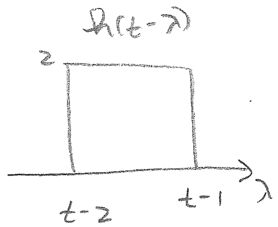
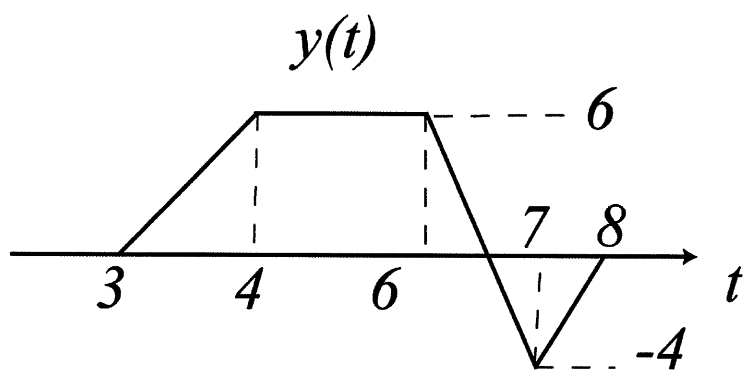
4) (20 points) An LTI system has impulse response, input, and output as shown below. Determine numerical values for the parameters *a, b, c, d* and *e*. *Note that the diagrams are not to scale!*

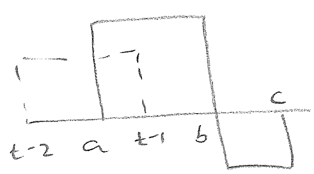






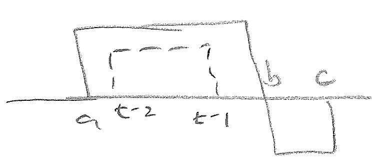


$t-1=a \quad t=a+1=3 \quad a=2$



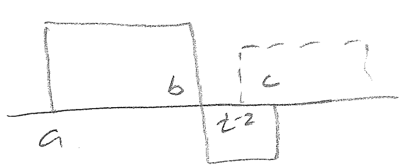
$6=(1)(2)(d) \quad d=3$

$t-1=b \quad t=b+1=6 \quad b=5$



$-4=(1)(2)(e) \quad e=-2$

$t-2=c \quad t=c+2=8 \quad \boxed{c=6}$



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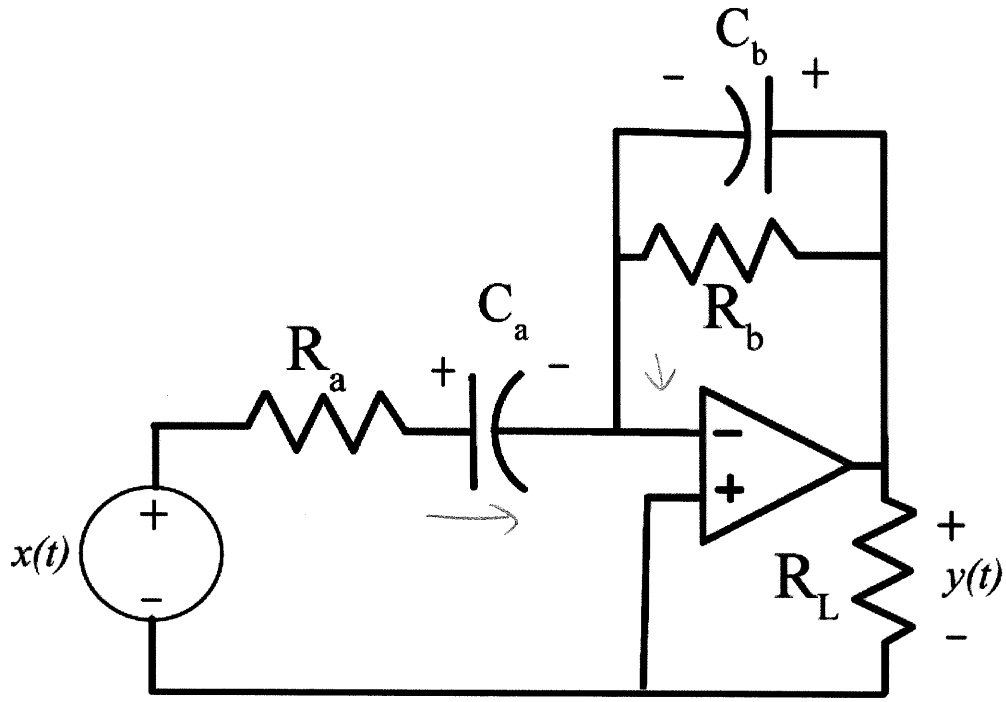
5) (15 points) Determine the transfer function for the following circuit in terms of the parameters given. For full credit you must simplify your result as much as possible.



$$\frac{X(s)}{R_a + \frac{1}{C_a s}} + \frac{Y(s)}{R_b} + \frac{Y(s)}{\frac{1}{C_b s}} = 0$$

$$\frac{X(s)\, C_a s}{R_a C_a s + 1} = -Y(s)\left[C_b s + 1/R_b\right] = -Y(s)\left[\frac{R_b C_b s + 1}{R_b}\right]$$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{R_b C_a s}{(R_a C_a s + 1)(R_b C_b s + 1)}}$$

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6) (15 points) Simplify the following expressions as much as possible. Do these problems in the time-domain.

a) $y(t) = e^{-2(t-1)}\delta(t-2)$

$$y(t) = e^{-2(2-1)}\delta(t-2) = \boxed{e^{-2}\delta(t-2) = y(t)}$$

b) $y(t) = e^{-2(t-1)} * \delta(t-2)$

$$y(t) = \int_{-\infty}^{\infty} e^{-2(\lambda-1)}\delta(t-\lambda-2)\,d\lambda = e^{-2((t-2)-1)}\int_{-\infty}^{\infty}\delta(t-\lambda-2)\,d\lambda$$

$$\boxed{y(t) = e^{-2(t-3)}}$$

c) $y(t) = \delta(t-1) * \delta(t-2)$

$$y(t) = \int_{-\infty}^{\infty}\delta(\lambda-1)\,\delta(t-\lambda-2)\,d\lambda = \delta(t-1-2)$$

$$= \boxed{\delta(t-3) = y(t)}$$

d) $y(t) = \int_{1}^{t-1} e^{-2(t-\lambda)}e^{-3\lambda}\,d\lambda$

$$= e^{-2t}\int_{1}^{t-1} e^{2\lambda}e^{-3\lambda}\,d\lambda = e^{-2t}\int_{1}^{t-1} e^{-\lambda}\,d\lambda$$

$$= e^{-2t}\left[-e^{-\lambda}\right]_{1}^{t-1} = e^{-2t}\left[e^{-1} - e^{-(t-1)}\right] = \boxed{e^{-2t-1} - e^{-3t+1} = y(t)}$$