

## ECE-205 : Dynamical Systems

### Homework #4

**Due :** Thursday January 7 at the beginning of class

1) For each of the following mathematical description of a system, determine if the system is linear, time invariant, causal, and memoryless and fill in the following table. Unless the system does not meet the homogeneity condition, you must use a formal technique to show the system is linear. If it does not meet the homogeneity condition, you must give an example. For the memoryless and causal you can just say it's obvious (assuming it is).

System	Linea (Y/N)	Time Invariant (Y/N)	Memoryless (Y/N)	Causal (Y/N)
$\dot{y}(t) = x(t)$				
$y(t) = \frac{1}{1+x(t)}$				
$\ddot{y}(t) + e^{-t}y(t) = \cos(t)x(t)$				
$y(t) = \int_{-\infty}^t e^{t+\lambda} x(\lambda) d\lambda$				
$y(t) = \int_{-\infty}^t e^{t-\lambda} x(\lambda+1) d\lambda$				
$y(t) = \int_{-\infty}^t e^{-\lambda} \lambda x(\lambda) d\lambda$				
$\dot{y}(t) + y(t)x(t) = x(t)$				

*Answers: 5 are L, 4 are TI, 1 is memoryless, 6 are causal*

2) For each of the following mathematical descriptions of a system, determine if the system is BIBO stable.

a)  $y(t) = \frac{1}{1+x(t)}$       b)  $y(t) = e^{-x(t)}$       c)  $y(t) = \cos(x(t))$

d)  $y(t) = \int_{-\infty}^t e^{-2(t-\lambda)} x(\lambda) d\lambda$     e)  $y(t) = \int_{-\infty}^t e^{2(t-\lambda)} x(\lambda) d\lambda$     f)  $y(t) = tx(t)$

*Answers: 3 are BIBO stable*

3) Assume we have an LTI system with the input-output relationship  $x(t) = u(t) \rightarrow y(t) = e^{-2(t-1)}u(t-1)$ . This means that if the input is a unit step starting at zero, the output is a decaying exponential starting at  $t = 1$ . Determine an expression for output the following inputs:

a)  $x(t) = u(t-2)$     b)  $x(t) = 6u(t+2)$     c)  $x(t) = 2[u(t-1) - u(t-2)]$

*The answer to c is  $y(t) = 2e^{2(t-2)}u(t-2) - 2e^{2(t-3)}u(t-3)$*

#### 4) Matlab/Prelab

One property of linear systems that we have not discussed is that *if a system is linear, then if the input is a sinusoid, then the output must be a sinusoid*. The magnitude and phase can be different between the input and output signals, but they must both be sinusoids of the same frequency or the system is not linear.

a) Create a new Matlab m-file. Use **linspace** to create a time vector **t** from 0 to 20 seconds using 2000 points. Create the input sinusoid **x = cos(t)**; You can just type this command into your m-file, you should not use an anonymous function here. The variables **t** and **x** will be an array of the form

$$t = [t_1 \ t_2 \dots t_{2000}] = [0 \dots 20] \quad x = [\cos(t_1) \ \cos(t_2) \dots \cos(t_{2000})]$$

The systems we want to look at are represented by the following relationships

system A:  $y = \tan(x)$    system B:  $y = \exp(x)$    system C :  $y = 1 / (2 + x)$    system D:  $y = x^2$

Use Matlab to plot the system input and system output on four graphs (one a single page). Use the **subplot** command. Before your first plot, type **orient tall**, this will use more of the page. Your graph should look like that on the next page. You will also need the commands **xlabel**, **ylabel**, **grid**, and **legend** (among others). You may also need to use the commands **./** and **.\*** for element by element division and multiplication. All of these systems are clearly non-linear. **Turn in your code and your plot.**

b) Sometimes, although a system is nonlinear, we can approximate the system as linear if we only consider **small signals**. If the input signal is small enough, we know from calculus that we can approximate the output using the Taylor series (about  $x = 0$ , since we assume a small input),

$$y(x) \approx y(0) + \left( \left. \frac{dy(x)}{dx} \right|_{x=0} \right) x$$

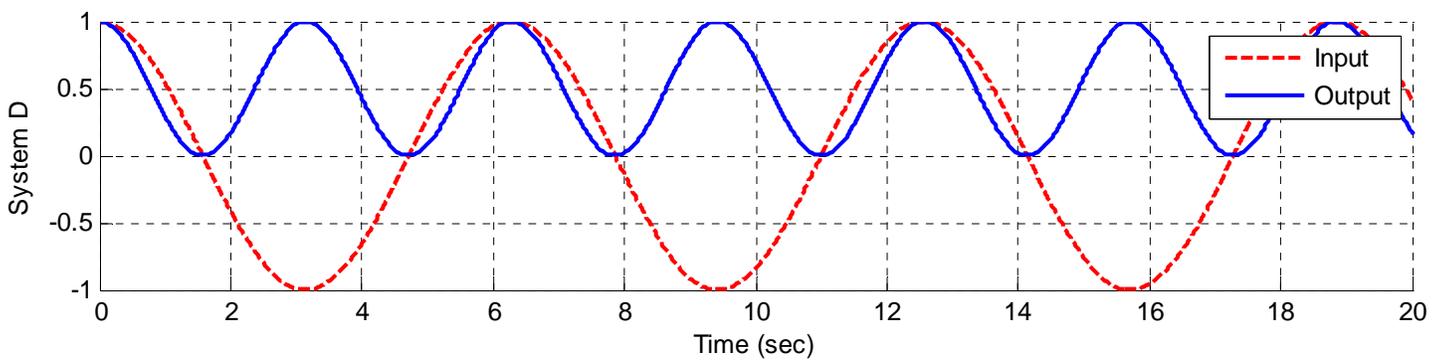
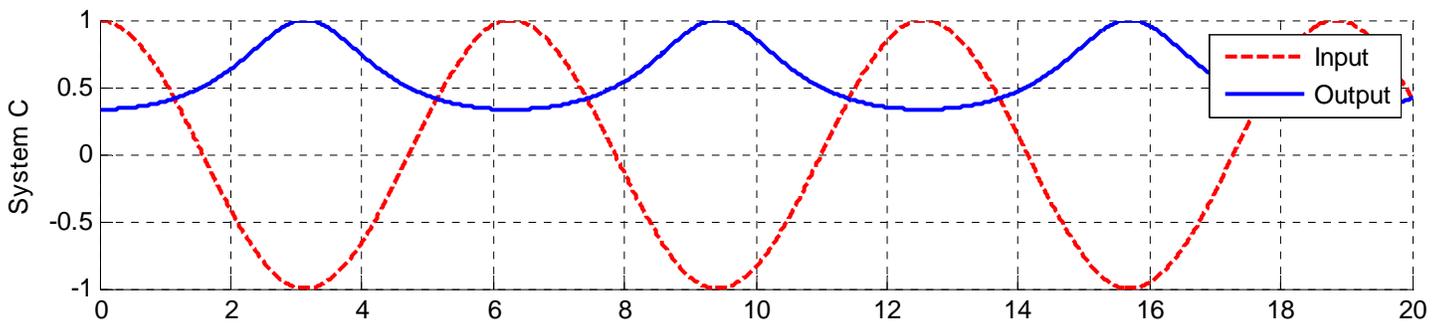
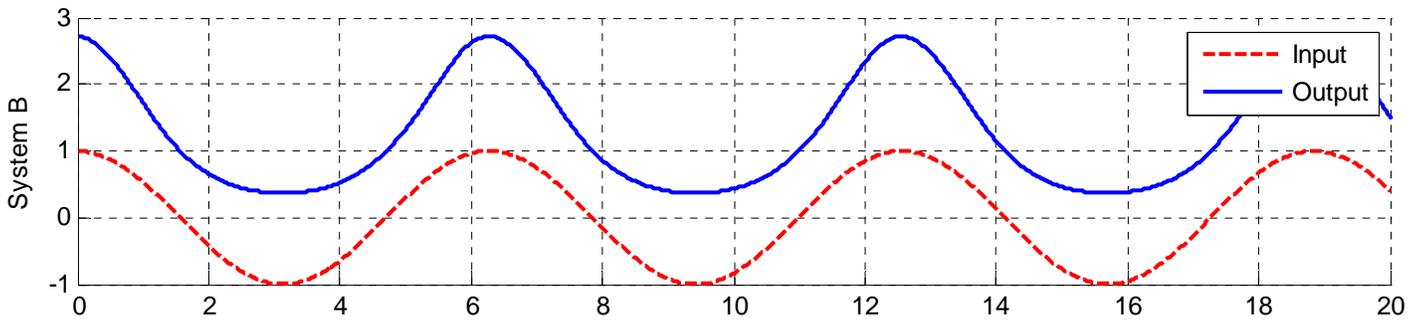
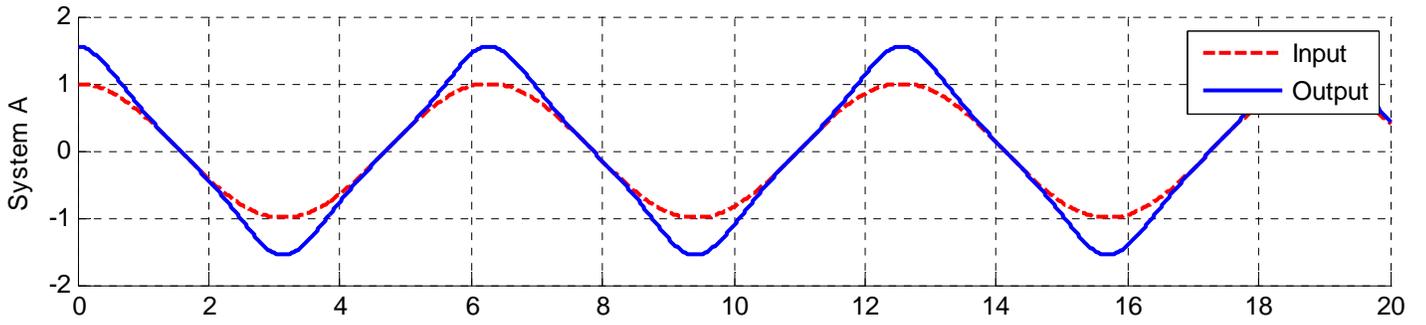
Even though  $x$  is a function of time, we can still use this approximation for functions. We can rewrite this as

$$\Delta y = y(x) - y(0) \approx mx = \left( \left. \frac{dy(x)}{dx} \right|_{x=0} \right) x$$

where

$$m = \left( \left. \frac{dy(x)}{dx} \right|_{x=0} \right)$$

If this first order approximation is valid, then we have the linear relationship  $\Delta y \approx mx$ . Now change your code so the input to the system is **x = cos(t)/10**. For each of your systems, determine  $y(0)$  and plot time (t) versus  $y - y(0)$  using the same format as in part a. Do not plot the input signal, we only want to see the output signal. The first three system should now approximately be linear, while the fourth is still non-linear. **Turn in your code and your plot.**



Time (sec)