Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/20
Problem 2 ________/15
Problem 3 ________/15
Problem 4 ________/15
Problems 5 ________/14
Problems 6-12 ________/21

Total ____________
1) **(20 points)** For the following transfer functions, determine both

- the **impulse response**
- the **unit step response**

*Do not forget any necessary unit step functions.*

a) \( H(s) = \frac{4}{s^2 + 4s + 8} \)

b) \( H(s) = \frac{se^{-2s}}{(s+1)^2} \)

\[ a) \quad H(t) = 2 \frac{2}{(t+2)^2 + 2^2} \]

\[ b) \quad \frac{u(t)}{s(s^2 + 4s + 4)} = A + B \left( \frac{2}{(t+2)^2 + 4} \right) + C \left( \frac{t+2}{(t+2)^2 + 4} \right) \]

\[ A = \frac{1}{2} \quad B = \frac{1}{2} \quad C = -\frac{1}{2} \]

\[ B = -1 \quad 0 = A + C \quad C = -\frac{1}{2} \]

\[ \frac{2}{t+2} = \frac{1}{2} = -\frac{1}{2} + \frac{B}{2} \quad -\frac{1}{4} = \frac{B}{2} \quad B = -\frac{1}{2} \]

\[ h(t) = \left[ \frac{1}{2} - \frac{1}{2} e^{-2t} \sin(2t) - \frac{1}{2} e^{-2t} \cos(2t) \right] u(t) \]

b) \( G(s) = \frac{B}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2} \)

\[ B = -1 \quad \ell \neq \pi \quad A = 4 \]

\[ g(t) = e^{-t} - te^{-t} \]

\[ h(t) = g(t-1) = \left[ e^{-(t-1)} - (t-1)e^{-(t-1)} \right] u(t-1) \]

\[ G(s) = \frac{A}{(s+1)^2} \quad g(t) = te^{-t} \]

\[ y(t) = g(t-1) = (t-1)e^{-(t-1)} \]
2) (15 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function \( G_p(s) = \frac{3}{s + 5} \)

![Block Diagram]

a) Determine the settling time of the plant alone (assuming there is no feedback)

\[ T_s = \frac{4}{5} \]

b) Determine the steady state error for plant alone assuming the input is a unit step (simplify your answer as much as possible)

\[ e_{ss} = \frac{1 - \frac{3}{5}}{\frac{2}{5}} = \frac{2}{5} = e_{ss} \]

c) For a proportional controller, \( G_c(s) = k_p \), determine the closed loop transfer function \( G_o(s) \)

\[ G_o(s) = \frac{2k_p}{s + s + 3k_p} \]

d) Determine the settling time of the closed loop system, in terms of \( k_p \)

\[ T_s = \frac{4}{5 + 3k_p} \]

e) Determine the steady state error of the closed loop system for a unit step, in terms of \( k_p \) (simplify your answer as much as possible)

\[ e_{ss} = \frac{1 - \frac{3k_p}{5 + 3k_p}}{\frac{5}{5 + 3k_p}} = \frac{5}{5 + 3k_p} = e_{ss} \]

f) For an integral controller, \( G_c(s) = \frac{k_i}{s} \), determine the closed loop transfer function \( G_o(s) \) and the steady state error for a unit step in terms of \( k_i \)

\[ G_o(s) = \frac{3k_i}{s^2 + 5s + 3k_i} \]

\[ e_{ss} = 0 \]
3) (15 points) For the following circuit

- Determine the ZIR
- Determine the ZSR
- Determine the transfer function

\[ I_{in}(t) = \frac{V_{out}(s)}{L \frac{d}{dt}} + \frac{i(i0)}{s} + \frac{V_{out}(s)}{R_b} = \frac{i(i0)}{s} + V_{out}(s) \left[ \frac{1}{L \frac{d}{dt}} + \frac{1}{R_b} \right] \]

\[ = \frac{i(i0)}{s} + V_{out}(s) \left[ \frac{R_b + L \frac{d}{dt}}{R_b L \frac{d}{dt}} \right] \]

\[ V_{out}(s) = \left[ \frac{R_b L \frac{d}{dt}}{L \frac{d}{dt} + R_b} \right] I_{in}(t) + \left[ \frac{-i(i0) R_b L}{L \frac{d}{dt} + R_b} \right] \]

\[ H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \frac{R_b L \frac{d}{dt}}{L \frac{d}{dt} + R_b} = H(i0) \]
4) (15 Points) Determine the transfer function for the following circuit.

\[ V^+ = \frac{\mathcal{V}_{in}(s)}{R_a + \frac{1}{C_a R_a}} = \frac{\mathcal{V}_{in}(s)}{R_a C_a s + 1} = V^- = \frac{\mathcal{V}_{out}^+(s)}{R_b + \frac{1}{C_b R_b}} = \frac{\mathcal{V}_{out}(s)}{R_b C_b s + 1} \]

\[ \frac{\mathcal{V}_{out}(s)}{\mathcal{V}_{in}(s)} = H(s) = \frac{R_b C_b s + 1}{R_a C_a s + 1} \]
5) (15 points) Consider a linear time invariant system with impulse response given by

\[ h(t) = e^{-(t+1)} u(t+1) \]

The input to the system is given by

\[ x(t) = e^{-t}[u(t) - u(t-1)] + 2u(t-2) \]

Using **graphical evaluation**, determine the output \( y(t) \). Specifically, you must

- Flip and slide \( h(t) \), **NOT** \( x(t) \)
- Show graphs displaying both \( h(t-\lambda) \) and \( x(\lambda) \) for each region of interest
- Determine the range of \( t \) for which each part of your solution is valid
- Set up any necessary integrals to compute \( y(t) \). Your integrals must be complete, in that they cannot contain the symbols \( x(\lambda) \) or \( h(t-\lambda) \) but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- **DO NOT EVALUATE THE INTEGRALS!!**
Problems 6 and 7 refer to the impulse responses of six different systems given below:

\[ h_1(t) = [1 + e^{-t}]u(t) \]
\[ h_2(t) = e^{-2t}u(t) \]
\[ h_3(t) = [2 + \sin(t)]u(t) \]
\[ h_4(t) = [1 - t^3 e^{-0.1t}]u(t) \]
\[ h_5(t) = [t \sin(t) + e^{-t}]u(t) \]
\[ h_6(t) = [te^{-t} \cos(5t) + e^{-2t} \sin(3t)]u(t) \]

6) The number of (asymptotically) **marginally stable systems** is  
   a) 0  b) 1  c) 2  d) 3

7) The number of (asymptotically) **unstable systems** is  
   a) 0  b) 1  c) 2  d) 3

8) Which of the following transfer functions represents a (asymptotically) **stable** system?

\[ G_a(s) = \frac{s - 1}{s + 1} \]
\[ G_b(s) = \frac{1}{s(s + 1)} \]
\[ G_c(s) = \frac{s}{s^2 - 1} \]
\[ G_d(s) = \frac{s + 1}{(s + 1) + 1(\pi + 1)} \]
\[ G_e(s) = \frac{(s - 1 - \pi)(s - 1 + \pi)}{s} \]
\[ G_f(s) = \frac{(s - 1 - \pi)(s - 1 + \pi)}{s + 1 - \pi(s + 1 + \pi)} \]

a) all but \( G_c \)  b) only \( G_a, G_b, \) and \( G_d \)  c) only \( G_a, G_d, \) and \( G_f \)

\[ \square \] only \( G_a, G_d, \) and \( G_f \)

d) only \( G_d \) and \( G_f \)  e) only \( G_a \) and \( G_d \)
For problems 9-12, consider the signal flow graph representation of the following block diagram.

9) How many paths are there?  
   a) 0  b) 1  c) 2  d) 3  e) 4

10) How many loops are there?  
    a) 0  b) 1  c) 2  d) 3  e) 4

11) The determinant (Δ) is  
     a) 1  b) 1 - H_2H_3H_4  c) 1 + H_2H_3H_4  d) none of these

12) The transfer function is  
     a) 1  b) \frac{H_5H_5 + H_1H_2H_5}{1 + H_2H_3H_4}  c) \frac{H_5H_5 + H_1H_2H_3}{1 - H_2H_3H_4}