Name Solutions Mailbox

ECE-205
Exam 2
Fall 2013

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/15

Problem 2 ________/17

Problem 3 ________/18

Problem 4 ________/24

Problem 5 ________/26

Total ____________

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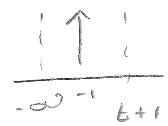
1) (15 points) Simplify the following as much as posible. Be sure to include any necessary unit step functions

$$y(t) = [t + e^t]\delta(t-1) = \boxed{[1+e]\,\delta(t-1)}$$

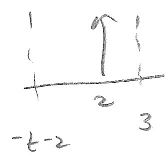
$$y(t) = \int_{-\infty}^{t+1} \delta(\lambda+1)d\lambda = \boxed{u(t+2)}$$



$t+1 > -1$

$t+2 > 0$

$$y(t) = \int_{-t-2}^{3} \delta(\lambda-2)d\lambda = \boxed{u(t+4)}$$



$-t-2 < 2$

$0 < t+4$

$$y(t) = \int_0^t e^{-(t-\lambda)}e^{-2\lambda}d\lambda = e^{-t}\int_0^t e^{-\lambda}d\lambda = e^{-t}\left[-e^{-\lambda}\Big|_0^t\right] = e^{-t}\left[1-e^{-t}\right]$$

$$= \boxed{e^{-t} - e^{-2t}}$$

$$y(t) = \int_0^t e^{-3(t-\lambda)}e^{-3\lambda}d\lambda = e^{-3t}\int_0^t d\lambda = \boxed{t\,e^{-3t}}$$

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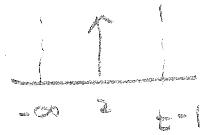
2) (17 Points) Determine the ***impulse response*** for the following systems. Don't forget any necessary unit step functions

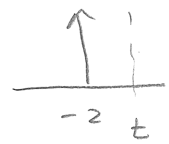


$t-1>2$

$t-3>0$

a) $\dot{y}(t) = x(t) + \int_{-\infty}^{t-1} x(\lambda - 2)d\lambda$

$$\boxed{h(t) = \delta(t) + u(t-3)}$$



$t>-2$

$t+2>0$

b) $y(t) = \int_{-\infty}^{t} e^{-(t-\lambda)} x(\lambda + 2)d\lambda$

$$\boxed{h(t) = e^{-(t+2)}u(t+2)}$$

c) $\dot{y}(t) + 2y(t) = 3x(t-1)$

$$\frac{d}{dt}\left[h(t)e^{2t}\right] = e^{2t}3\,\delta(t-1) = e^{2}3\,\delta(t-1)$$

$$h(t)e^{2t} = 3e^{2}\int_{-\infty}^{t}\delta(\lambda-1)d\lambda = 3e^{2}u(t-1)$$

$$\boxed{h(t) = 3e^{-2(t-1)}u(t-1)}$$

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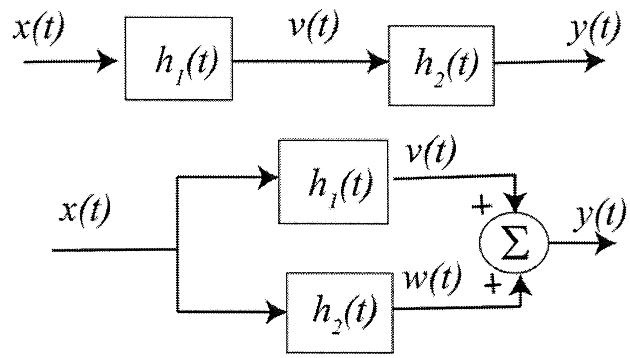
3) (18 points) For the following block diagram

For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input *x(t)* and output *y(t)*) and

ii) determine if the system is causal.



a) $h_1(t) = \delta(t),\ h_2(t) = \delta(t+2)$

b) $h_1(t) = \mathrm{e}^{-t}u(t),\ h_2(t) = u(t-2) + \delta(t-2)$

Series Connections:

(a) $h(t) = \int_{-\infty}^{\infty} \delta(t-\lambda)\,\delta(\lambda+2)\,d\lambda = \boxed{\delta(t+2) = h(t)}$ $\boxed{\text{not causal}}$

(b) $h(t) = \int_{-\infty}^{\infty} e^{-\lambda} u(\lambda)\, u(t-\lambda-2)\,d\lambda + e^{-(t-2)}u(t-2)$

$= \int_{0}^{t-2} e^{-\lambda}\,d\lambda + e^{-(t-2)}u(t-2) = \left[1 - e^{-(t-2)} + e^{-(t-2)}\right]u(t-2)$

$\boxed{h(t) = u(t-2)}$ $\boxed{\text{causal}}$

Parallel Connections:

(a) $\boxed{h(t) = \delta(t) + \delta(t+2)}$ $\boxed{\text{not causal}}$

(b) $\boxed{h(t) = e^{-t}u(t) + u(t-2) + \delta(t-2)}$ $\boxed{\text{causal}}$

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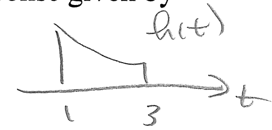
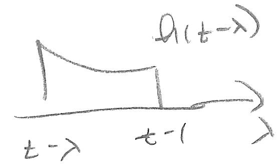
4) (24 points) Consider a linear time invariant system with impulse response given by

$$h(t) = e^{-t}[u(t-1) - u(t-3)]$$



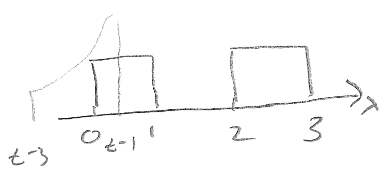
The input to the system is given by two rectangular pulses, given by

$$x(t) = [u(t) - u(t-1)] + [u(t-2) - u(t-3)]$$

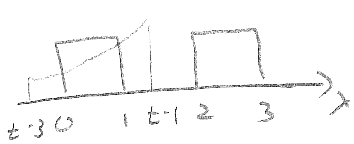


Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

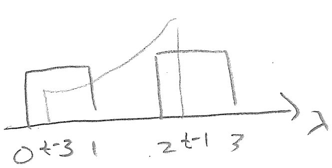
- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals **cannot contain any unit step functions**
- ***DO NOT EVALUATE THE INTEGRALS!!***



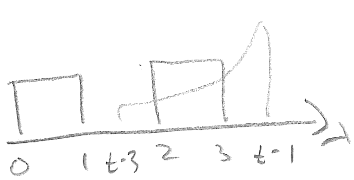
$1 \le t \le 2 \quad y(t) = \int_0^{t-1} e^{-(t-\lambda)} d\lambda$



$2 \le t \le 3 \quad y(t) = \int_0^1 e^{-(t-\lambda)} d\lambda$



$3 \le t \le 4 \quad y(t) = \int_{t-3}^1 e^{-(t-\lambda)} d\lambda + \int_2^{t-1} e^{-(t-\lambda)} d\lambda$



$4 \le t \le 5 \quad y(t) = \int_2^3 e^{-(t-\lambda)} d\lambda$

$5 \le t \le 6 \quad y(t) = \int_{t-3}^3 e^{-(t-\lambda)} d\lambda$

$y(t) = 0$ for $t \le 1$ or $t \ge 6$

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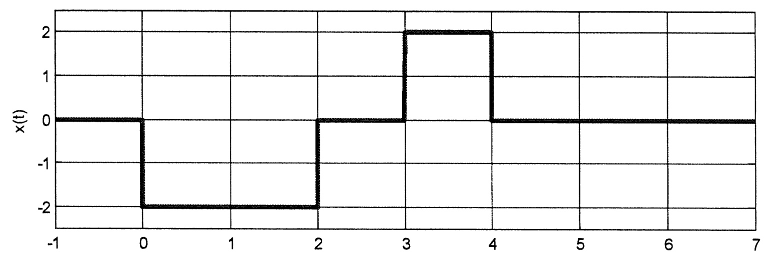
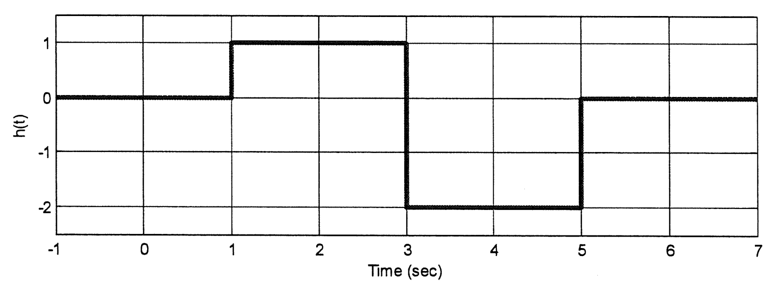
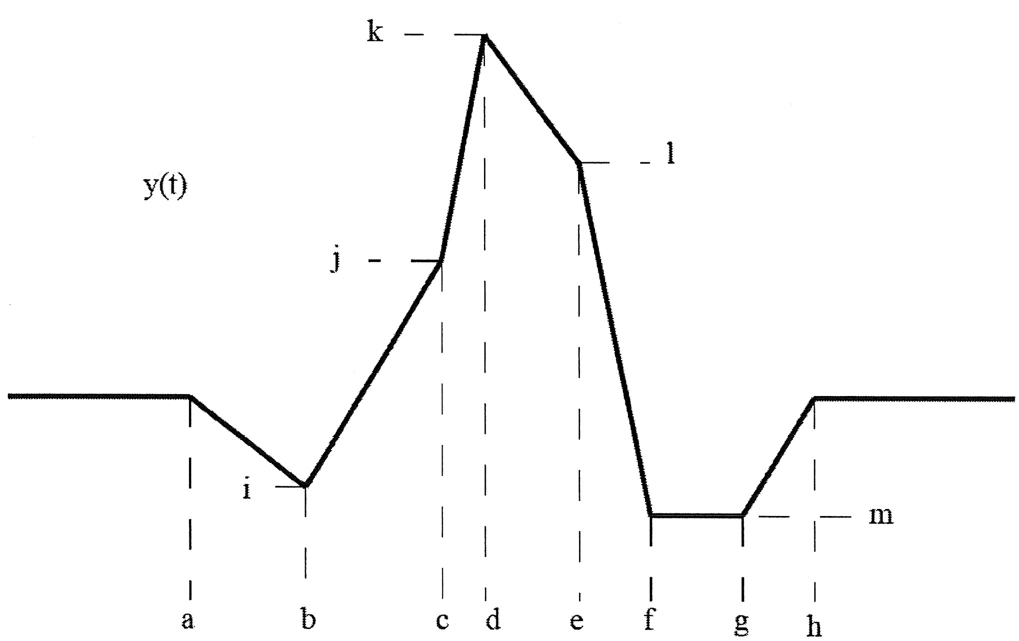
5) (26 Points) An LTI system has input, impulse response, and output as shown below. Determine numerical values for the parameters *a-m.* Note that parameters *a-h* correspond to *times*, and *i-m* correspond to *amplitudes.*

Hints:

- *Note that the output is not drawn to scale, it just represents the general shape of the output.*
- *A good way to check your answer is to flip and slide one of them, then flip and slide the other one.*
- *It is probably much easier to get the times correct than the amplitudes.*







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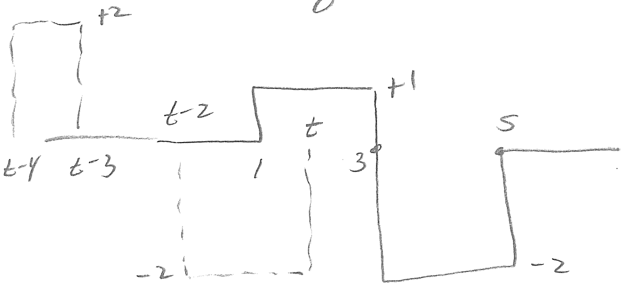
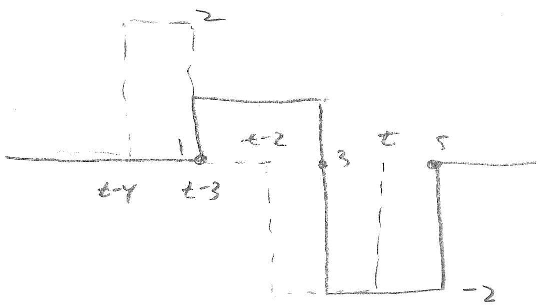
Flipping $x(t)$



overlap when $t = 1 = a$

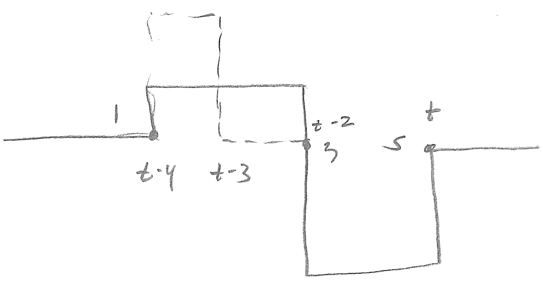
first maximum negative when $t = 3 = b$

$i = (-2)(1)(2) = -4 = i$



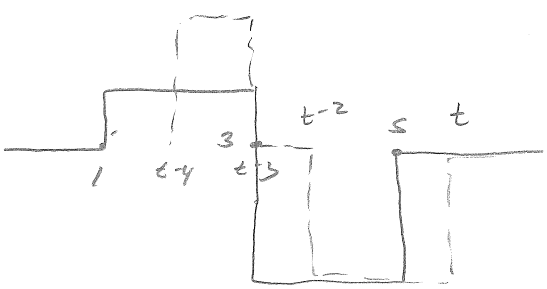
point c is when $t-3 = 1$ $t = 4 = c$

amplitude $j = (-2)(-2)(1) + (-2)(1)(1) = 2 = j$



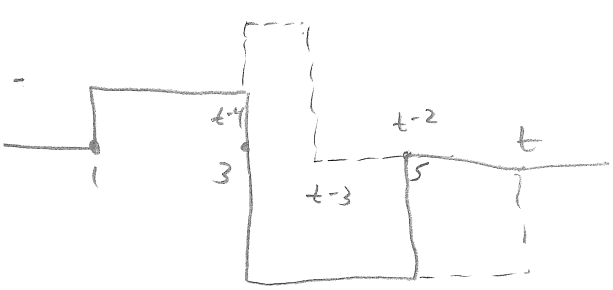
the maximum occurs when $t = 5 = d$

amplitude $K = (-2)(-2)(2) + (2)(1)(1) = 10 = K$



point e occurs when $t-3 = 3$ $t = 6 = e$

amplitude $l = (-2)(-2)(1) + (2)(1)(1) = 6 = l$



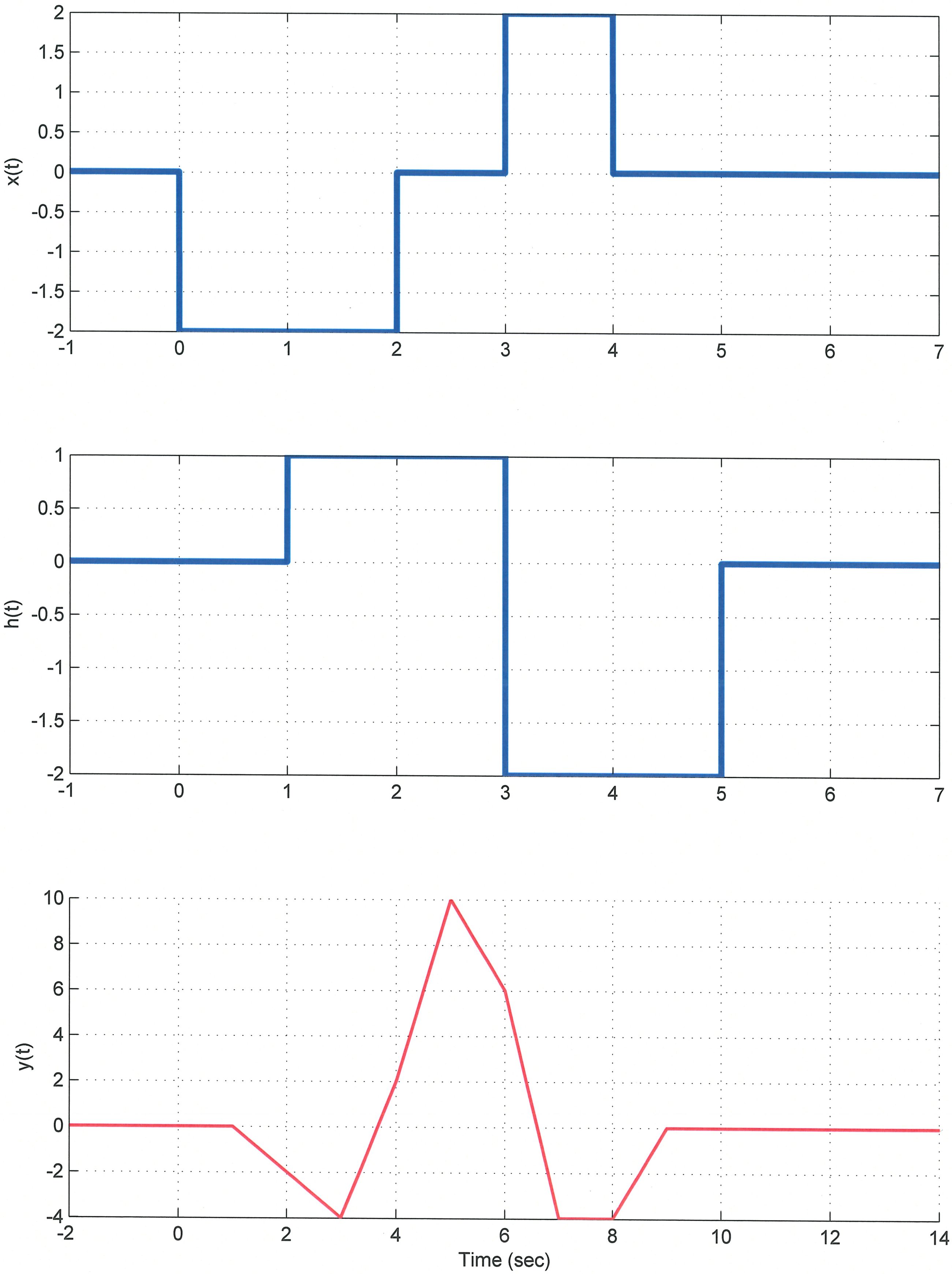
point f occurs when $t-4 = 3$ $t = 7 = f$

or $t-2 = 5$ $t = 7 = f$

amplitude $m = (2)(-2)(1) = -4 = m$

point g occurs when $t-3 = 5$ $t = 8 = g$

point h occurs when $t-4 = 5$ $t = 9 = h$


x(t)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
0
1
2
3
4
5
6
7
h(t)
1
0.5
0
-0.5
-1
-1.5
-2
-1
0
1
2
3
4
5
6
7
y(t)
10
8
6
4
2
0
-2
-4
-2
0
2
4
6
8
10
12
14
Time (sec)