ECE 130 HW#5 – Due Tuesday, March 23

1. Consider the following truth table where the four-bit number A (A3, A2, A1, A0) is input and the four-bit number X (X3, X2, X1, X0) is output:

A3	A2	A1	A0	X3	X2	X1	X0
0	*0*	*0*	*0*	1	1	1	1
0	*0*	*0*	*1*	1	1	0	0
0	*0*	*1*	*0*	1	0	0	1
0	*0*	*1*	*1*	0	1	1	0
0	*1*	*0*	*0*	0	1	0	1
0	*1*	*0*	*1*	0	0	1	1
0	*1*	*1*	*0*	0	0	1	0
0	*1*	*1*	*1*	0	0	1	0
1	*0*	*0*	*0*	0	0	0	1
1	*0*	*0*	*1*	0	0	0	1
1	*0*	*1*	*0*	0	0	0	1
1	*0*	*1*	*1*	0	0	0	1
1	*1*	*0*	*0*	0	0	0	0
1	*1*	*0*	*1*	0	0	0	0
1	*1*	*1*	*0*	0	0	0	0
1	*1*	*1*	*1*	0	0	0	0

a. Write down the correct functional representation for each of the four outputs in the canonical sigma notation:

X3 (A3, A2, A1, A0) = Σ(0, 1, 2)

X2 (A3, A2, A1, A0) = Σ(0, 1, 3, 4)

X1 (A3, A2, A1, A0) = Σ(0, 3, 5, 6, 7)

X0 (A3, A2, A1, A0) = Σ(0, 2, 4, 5, 8, 9, 10, 11)

b. Write down the correct functional representation for each of the four outputs in the canonical pi notation:

X3 (A3, A2, A1, A0) = Π(3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)

X2 (A3, A2, A1, A0) = Π (2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)

X1 (A3, A2, A1, A0) = Π (1, 2, 4, 8, 9, 10, 11, 12, 13, 14, 15)

X0 (A3, A2, A1, A0) = Π (1, 3, 6, 7, 12, 13, 14, 15)

c. Expand the canonical sigma notation for X3 into a canonical sum-of-products equation (do not attempt to simplify)

$X3 = A3' A2' A1' A0' + A3' A2' A1' A0 + A3' A2' A1 A0'$

d. Expand the canonical pi notation for X0 into a canonical product-of-sums equation (do not attempt to simplify)

$X0 = (A3+A2+A1'+A0)(A3+A2+A1'+A0')(A3+A2'+A1'+A0)(A3+A2'+A1'+A0')$
$(A3'+A2'+A1+A0)(A3'+A2'+A1+A0')(A3'+A2'+A1'+A0)(A3'+A2'+A1'+A0')$

e. Using Boolean algebra, simplify the canonical sum-of-products equation for X3 (from part c) into a sum-of-products equation that has the fewest possible literals and operators. (Note that there may be a more simplified form that is *not* sum-of-products.)

$X3 = A3' A2' A1' A0' + A3' A2' A1' A0 + A3' A2' A1 A0'$

using T7: $(A3' A2' A1' A0' + A3' A2' A1' A0) + A3' A2' A1 A0'$
using T8: $(A3' A2' A1' (A0' + A0)) + A3' A2' A1 A0'$
using T5: $(A3' A2' A1' (1)) + A3' A2' A1 A0'$
using T1: $(A3' A2' A1') + A3' A2' A1 A0'$
using T7: $A3' A2' A1' + A3' A2' A1 A0'$

according to k-map:

A1A0\A3A2	00	01	11	10
00	1	0	0	0
01	1	0	0	0
11	0	0	0	0
10	1	0	0	0

2. Prove Theorem Nine …

A * (A + B) = A

a. … using only other theorems (justify each step with the theorem number, *like T1*):

A * (A + B)

T8: A*A + A*B
T3: A + A*B
T1: A*1 + A*B
T8: A*(1 + B)
T2: A*(1)
T1: A

b. … using only postulates

A	B	X=A+B	A*X	A
0	0	0	0	0
0	1	1	0	0
1	0	1	1	1
1	1	1	1	1

According to postulate 1, A/B can only have two values, 1 and 0. Hence, there are 4 possible table entries corresponding to the 4 possible combinations of A and B. Postulates 3, 4, and 5 define the behavior for the + and * operators for the 3^{rd} and 4^{th} column. For every possible case of A and B, the expression A*(A+B) is equivalent to the value of A. Therefore, by perfect induction, A = A*(A+B).