

Bandgap as a Function of Temperature

The bandgap, "Eg", of a semiconductor or insulator is the energy difference between the top of its valence band and the bottom of its conduction. The bandgap is a forbidden energy range between the valence and conduction bands, i.e., electrons can not rest anywhere within the band gap. An electron in the valence band must absorb enough energy to jump across the band gap in order to be in the conduction band and participate in conduction.

The bandgap of a semiconductor <u>decreases as temperature increases</u>. An increase in temperature causes the atoms to have a higher thermal energy, which result in greater atomic vibrations. The greater atomic vibrations, in turn, tend to increase the distances between the atoms, resulting in lattice expansion. This increase in inter-atomic separations tends to lower the potential seen by the electrons of the atoms, reducing the bandgap. Another way of looking at this is that increasing the distances between atoms 'weakens' the bonds between them, and therefore decreases the additional energy needed to break the bonds and release electrons from the valence to the conduction band.

The application of a high stress on a semiconductor also affects inter-atomic spacing, and can therefore affect the bandgap too. Compressive stress, which tends to decrease inter-atomic spacing, increases the bandgap while tensile stress, which tends to increase inter-atomic spacing, decreases the band gap.

The dependence of the energy band gap on temperature may be expressed mathematically as follows:

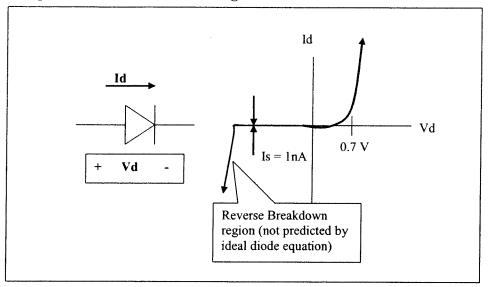
$$Eg(T) = Eg(0) - [(\alpha T^2)/(T+\beta)]$$

where Eg(0) is the limiting value of the band gap at absolute zero while α and β are constants chosen to fit experimental data. Table 1 shows the empirical a and β values for Ge, Si, and GaAs.

Table 1. α and β Values for Various Semiconductors*

	_		
		اِ	
2.41 X 10			
	4 73 × 10-4	4.77 x 10 ⁻⁴	a [eV/K]
1.512	1		
-	1.166	0.7437	$E_g(0)$ [eV]
GaA:	Silicon	Germanium	

Temperature Measurement Using Forward-Biased Semiconductor PN Junction



The Boltzmann diode equation relates the voltage across a diode (Vd) to the current through it (Id). Assuming a junction grading coefficient n = 1, the diode equation becomes:

$$Id = I_{S} \cdot e^{\frac{Vd}{\left(\frac{k \cdot T}{q}\right)}} - 1$$

Where:

Is is the "reverse saturation current". Is is on the order of 1 nA (it is due to thermal generation of electron-hole pairs in the depletion region of the pn junction, and is therefore a strong function of temperature.

k is Boltzmann's constant ($k = 1.3810^{-23}$ J/K)

q is the magnitude of the charge on an electron = $q = 1.60210^{-19}$ Coulombs T is the temperature in degrees Kelvin = degrees Centigrade + 273.15 degrees.

Note that at room temperature T := 25 C = T := 27 + 273.15 KT = 300.15 K

At T = 300 K, the average thermal energy per Coulomb is $\frac{k \cdot T}{q}$ = 0.026 Volts

For the case when the diode is forward biased to the point that Id = 1 mA, it is clear that the exponential term must dominate, and the "1" can be ignored, thus this forward-bias approximation implies that

$$Id = I_{c} \cdot e^{\frac{Vd}{\left(\frac{k \cdot T}{q}\right)}}$$
 (1)

In addition, the reverse saturation current "Is" is itself a strong function of temperature, sin "Is" is due to thermal generation of electron-hole pairs in the depletion region. Thus "Is" is proportional to the probability of a Si atom breaking up into an electron-hole pair due to thermal agitation. This probability can be shown by "Boltzmann statistics" to be exp(-(Bandgap Energy) / (Average Thermal Energy per electron)). The average thermal energy per electron is given by kT. Thus "Is" is given by

$$Is = K \cdot e^{\left(\frac{-Eg}{k \cdot T}\right)}$$
 (2)

Where Eg is the band gap of the semiconductor in eV. For Si, Eg = 1.1 eV.

Substituting (1) into (2) yields

$$Id = K \cdot e^{\left(\frac{-Eg}{k \cdot T}\right)} \cdot e^{\left(\frac{k \cdot T}{q}\right)} = K \cdot e^{\left(\frac{-Eg + q \cdot Vd}{k \cdot T}\right)}$$

Solving for Vd

$$Vd = \frac{Eg}{a} - \frac{k \cdot T}{a} \cdot (\ln(K) - \ln(Id))$$
 (3)

Note that if the diode current "Id" is held constant by connecting the diode in the forward direction across a constant current source, then Vd is linear with respect to temperature T

To investigate the sensitivity of the forward-biased pn junction, take the derivative of (3) w.r.t. T

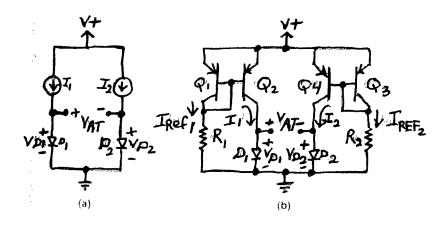
$$\frac{dVd}{dt} = \frac{k}{q} \cdot (\ln(K) - \ln(Id))$$

For a silicon pn junction diode that is forward biased to a constant current of Id = 1 mA, in the vicinity of room temperature (around 300 Kelvin) we have

$$\frac{\text{dVd}}{\text{dt}} = -2.0 \frac{\text{mV}}{\text{degree}} \tag{4}$$

Because Id appears inside a natural logarithm function, Id can vary by an order of magnituabove and below 1 mA, and the temperature sensitivity will not stray very far from -2.0 mV/degree!

An improved pn junction linear temperature sensor using OP AMPs



The temperature sensor now consists of two matched (same value of Is) forward-biased pn junction diodes. Both diodes are at the Kelvin temperature that is to be measured (T). Diode D1 passes current I_1 and diode D2 passes current I_2 . These currents are generated by two "current mirror" current source circuits formed by the matched BJT transistor pairs: Q1 - Q2 and Q3 - Q4, where

$$I_1 = I_{REF1} = (V^+ - 0.7 V) / R1$$

 $I_2 = I_{REF2} = (V^+ - 0.7 V) / R2$.

From the Boltzmann Diode Equation (forward-bias approximation)

$$V_{d1} = \frac{kT}{q} \ln \left(\frac{I_1}{I_s} \right)$$
 and $V_{d2} = \frac{kT}{q} \ln \left(\frac{I_2}{I_s} \right)$

The differential output voltage from the above circuit is

$$V_{AT} = V_{d1} - V_{d2} = \frac{kT}{q} \cdot \ln\left(\frac{I_1}{I_s}\right) - \frac{kT}{q} \cdot \ln\left(\frac{I_2}{I_s}\right) = \frac{k \cdot T}{q} \cdot \ln\left(\frac{\frac{I_1}{I_s}}{\frac{I_2}{I_s}}\right) = \frac{k \cdot T}{q} \cdot \ln\left(\frac{I_1}{I_2}\right)$$

Therefore we have a linear relationship between the differential output voltage V_{AT} and Kelvin Temperature "T", where the temperature dependent effects of Is have been subtracted (cancelled) out!

Let us imagine that we could like a 0 - 5 V output from this sensor, such that 0 V corresponds to 0 degrees Centigrade, and 5 V corresponds to 50 degrees Centigrade.

Recall that the relationship between degrees Kelvin and degrees Centigrade is:

$$T = T_C + 273.15$$

If we pick R_1 and R_2 such that $I_1/I_2 = 5$, then

$$V_{AT} = \frac{k \cdot T}{q} \cdot \ln \left(\frac{I_1}{I_2} \right) = \frac{1.38 \cdot 10^{-23} \cdot T}{1.6 \cdot 10^{-19}} \cdot \ln(5)$$

Simplifying and replacing Kelvin temperature (T) with Centigrade temperature (T_C)

$$V_{AT}(T_C) := 1.388 \cdot 10^{-4} \cdot (T_C + 273.15)$$

At 0 degrees C At 50 degrees C $V_{AT}(0) = 0.038$ $V_{AT}(50) = 0.045$

Therefore, we must use an OP AMP subtracting circuit to subtract 0.038 V dc offset from VAT, and then amplify the result by $\frac{5}{(0.045-0.038)} = 714.286$

This produces the desired output voltage $Vo = 714.286(V_{AT} - 0.038)$

which varies between 0 V and 5 V as the temperature varies from 0 degrees C up to 50 degrees C.

Planck's Radiation Law relates spectral density of electromagnetic radiation from a blackbody (an ideal absorber of radiation) to its temperature above absolute zero in degrees Kelvin. $W_{\lambda}d\lambda$ represents the radiant power radiated per unit area (Watts/cm²) over a differential band of wavelengths, $d\lambda$, that is centered on a specific wavelength λ . Multiplying this law times an "emissivity constant" ϵ that represents the deviation in behavior from that of a blackbody (where $0 < \epsilon < 1$), allows this law to predict the electromagnetic radiation from any type of body. Human skin exhibits an emissivity of $\epsilon = 0.98$ (regardless of skini pigmentation). Tungsten exhibits an emissivity of $\epsilon = 0.4$.

$$W_{\lambda} = \frac{\varepsilon \cdot 3.74 \cdot 10^4}{\lambda^5 \left(\exp\left(\frac{1.44 \cdot 10^4}{\lambda \cdot T}\right) - 1.\right)} \qquad \left[\frac{W}{\text{cm}^2}\right]$$
 (2.25 Text)

Finding the value of λ ($\lambda_{m})$ at which \boldsymbol{W}_{λ} is maximum:

$$\frac{\mathrm{d}}{\mathrm{d}\lambda} W_{\lambda} = \frac{\mathrm{d}}{\mathrm{d}\lambda} \frac{\varepsilon \cdot 3.74 \cdot 10^4}{\lambda^5 \cdot \left(\exp\left(\frac{1.44 \cdot 10^4}{\lambda \cdot \mathrm{T}}\right) - 1.\right)} = 0$$
(Solving for $\lambda = \lambda_{\mathrm{m}}$) $\lambda_{\mathrm{m}} = \frac{2898}{\mathrm{T}} \quad (\mu \mathrm{m})$ (2.26 Text)

Thus the wavelength corresponding to the peak electromagnetic radiation from an object is inversely related to its temperature.

Note at T = 300 K (room temp)
$$\lambda_m = \frac{2898}{300} = 9.66 \cdot \mu m$$
 At T = 325 K (body temperature)
$$\lambda_m := \frac{2898}{325} \qquad \lambda_m = 8.917 \quad \mu m$$

The human eye responds to wavelengths between 0.4 μ m (violet) and 0.7 μ m (red) wavelengths. Thus the radiation from an object at room temperature is concentrated in the nonvisible "far infrared" portion of the electromagnetic spectrum. Thus the human eye cannot "see" the radiation emitted by an object at room temperature or the human body.

We may integrate to find the radiant power density between the wavelengths $\lambda 1$ and $\lambda 2$,

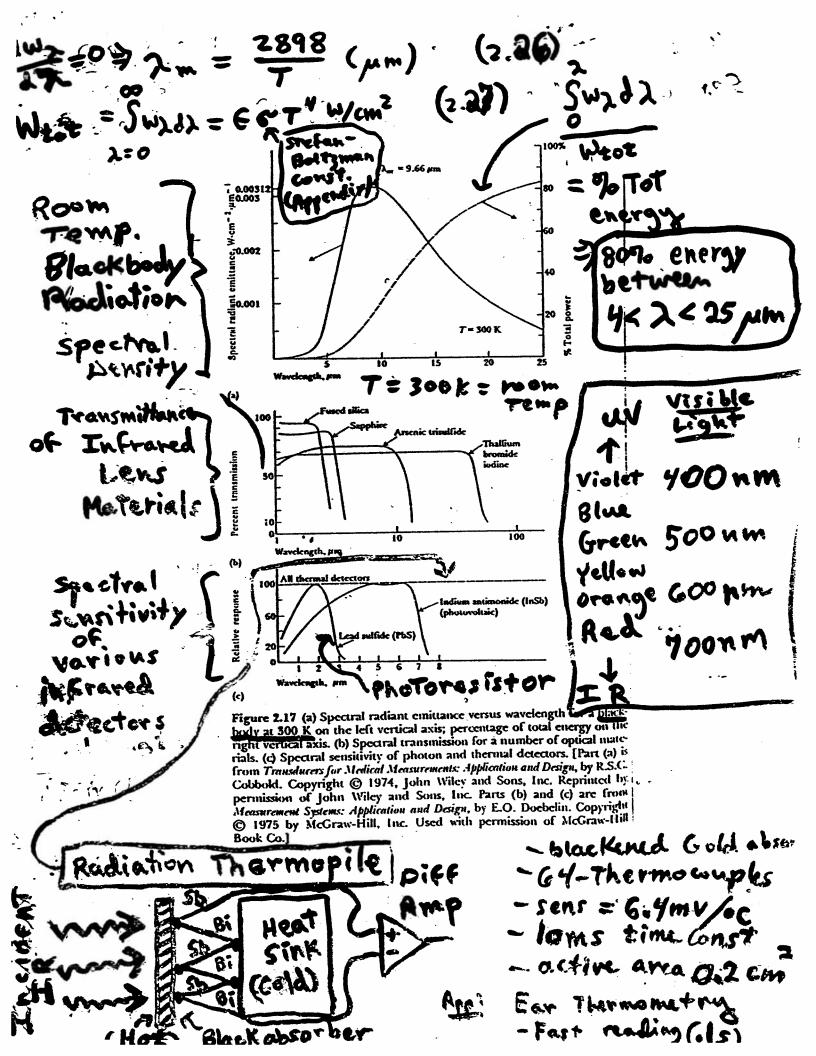
$$W_{\text{radiated}} = \int_{\lambda_1}^{\lambda_2} W_{\lambda} d\lambda \qquad \frac{W}{\text{cm}^2}$$

Equation (2.25) may be integrated with $\lambda 1 = 0$ and $\lambda 2 = \text{inifinity to find}$ the total radiated power density at all wavelengths:

$$W_{\text{tot.}} = \int_0^\infty W_{\lambda} d\lambda = \int_0^\infty \frac{\epsilon \cdot 3.74 \cdot 10^4}{\lambda^5 \left(\exp\left(\frac{1.44 \cdot 10^4}{\lambda \cdot T}\right) - 1.\right)} d\lambda = \epsilon \cdot \sigma \cdot T^4 \frac{W}{\text{cm}^2}$$
(2.27)

where σ is the Stephan-Boltzmann constant, 5.67X10⁻¹² W/(cm²*K⁴)

Thus the overall radiation intensity is proportional to T4, and so a simple wide-band thermopile electromagnetic radiation detector like the one shown below can be used to determine the temperature of an object without coming into contact with it (Ex: in-the-ear (tympanic membrane) thermometry).



Introduction to Infrared Pyrometers

Why should I use an infrared pyrometer to measure temperature in my application?

Infrared pyrometers allow users to measure temperature in applications where conventional sensors cannot be employed. Specifically, in cases dealing with moving objects (*i.e.*, rollers, moving machinery, or a conveyer belt), or where non-contact measurements are required because of contamination or hazardous reasons (such as high voltage), where distances are too great, or where the temperatures to be measured are too high for thermocouples or other contact sensors.

What should I consider about my application when selecting an infrared pyrometer?

The critical considerations for any infrared pyrometer include field of view (target size and distance), type of surface being measured (emissivity considerations), spectral response (for atmospheric effects or transmission through surfaces), temperature range and mounting (handheld portable or fixed mount). Other considerations include response time, environment, mounting limitations, viewing port or window applications, and desired signal processing.

FIELD OF VIEW

What is meant by Field of View, and why is it important?

The field of view is the angle of vision at which the instrument operates, and is determined by the optics of the unit. To obtain an accurate temperature reading, the target being measured should completely fill the field of view of the instrument. Since the infrared device determines the average temperature of all surfaces within the field of view, if the background temperature is different from the object temperature, a measurement error can occur (figure 1).

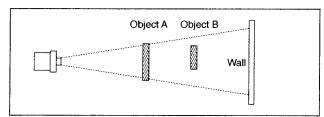


Figure 1: Field of view

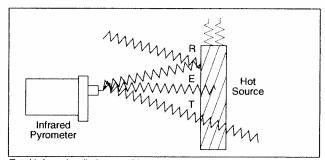
Most general purpose indicators have a focal distance between 20 and 60". The focal distance is the point at which the minimum measurement spot occurs. For example, a unit with a distance-to-spot size ratio of 120:1, and a focal length of 60" will have a minimum spot size of 0.5" at 60" distance. Close-focus instruments have a typical 0.1 to 12" focal length, while long-range units can use focal distances on the order of 50°. Many instruments used for long distances or small spot sizes also include sighting scopes for improved focusing. Field of view diagrams are available for most instruments to help estimate spot size at specific distances.

EMISSIVITY

What is emissivity, and how is it related to infrared temperature measurements?

Emissivity is defined as the ratio of the energy radiated by an object at a given temperature to the energy emitted by a perfect radiator, or blackbody, at the same temperature. The emissivity of a blackbody is 1.0. All values of emissivity fall between 0.0 and 1.0. Emissivity (ε) , a major but not uncontrollable factor in IR temperature measurement, cannot be ignored. Related to emissivity are reflectivity (R), a measure of an object's ability to reflect infrared energy, and transmissivity (T), a measure of an object's ability to pass or transmit IR energy. All radiation energy must be either emitted (E) due to the temperature of the body, transmitted (T) or reflected (R). The total energy, the sum of emissivity, transmissivity and reflectivity is equal to 1:

$$E + T + R = 1.0$$



Total infrared radiation reaching pyrometers

The ideal surface for infrared measurements is a perfect radiator, or a blackbody with an emissivity of 1.0. Most objects, however, are not perfect radiators, but will reflect and/or transmit a portion of the energy. Most instruments have the ability to compensate for different emissivity values, for different materials. In general, the higher the emissivity of an object, the easier it is to obtain an accurate temperature measurement using infrared. Objects with very low emissivities (below 0.2) can be difficult applications. Some polished, shiny metallic surfaces, such as aluminum, are so reflective in the infrared that accurate temperature measurements are not always possible.

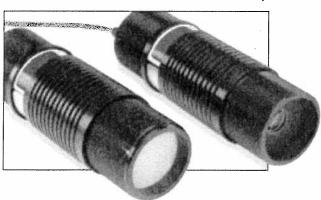


Reflectivity is usually a more important consideration than transmission except in a few special applications, such as thin film plastics. The emissivity of most organic substances (wood, cloth, plastics, etc.) is approximately 0.95. Most rough or painted surfaces also have fairly high emissivity values.

FIVE WAYS TO DETERMINE EMISSIVITY

There are five ways to determine the emissivity of the material, to ensure accurate temperature measurements:

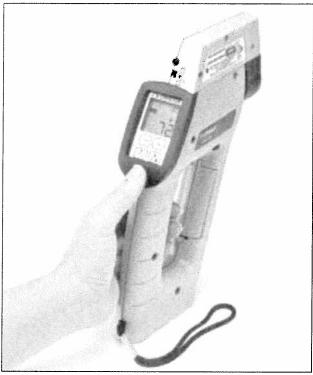
- 1. Heat a sample of the material to a known temperature, using a precise sensor, and measure the temperature using the IR instrument. Then adjust the emissivity value to force the indicator to display the correct temperature.
- 2. For relatively low temperatures (up to 500°F), a piece of masking tape, with an emissivity of 0.95, can be measured. Then adjust the emissivity value to force the indicator to display the correct temperature of the material.
- 3. For high temperature measurements, a hole (depth of which is at least 6 times the diameter) can be drilled into the object. This hole acts as a blackbody with emissivity of 1.0. Measure the temperature in the hole, then adjust the emissivity to force the indicator to display the correct temperature of the material.
- 4. If the material, or a portion of it, can be coated, a dull black paint will have an emissivity of approx. 1.0. Measure the temperature of the paint, then adjust the emissivity to force the indicator to display the correct temperature.
- 5. Standardized emissivity values for most materials are available (see pages 114-115). These can be entered into the instrument to estimate the material's emissivity value.



SPECTRAL RESPONSE

What is spectral response, and how will it affect my readings?

The spectral response of the unit is the width of the infrared spectrum covered. Most general purpose units (for temperatures below 1000°F) use a wideband filter in the 8 to 14 micron range. This range is preferred for most measurements, as it will allow measurements to be taken without the atmospheric interference (where the atmospheric temperature affects the readings of the instrument). Some units use wider filters such as 8 to 20 microns, which can be used for close measurements, but are "distance-sensitive" against longer distances. For special purposes, very narrow bands may be chosen. These can be used for higher temperatures, and for penetrations of atmosphere, flames, and gases. Typical low band filters are at 2.2 or 3.8 microns. High temperatures above 1500°F are usually measured with 2.1 to 2.3 micron filters. Other bandwidths that can be used are 0.78 to 1.06 for high temperatures, 7.9 or 3.43 for limited transmissions through thin film plastics, and 3.8 microns to penetrate through clean flames with minimum interference.



TEMPERATURE MEASUREMENT THROUGH GLASS

I want to measure the temperature through a glass or quartz window; what special considerations are there?

Transmission of the infrared energy through glass or quartz is an important factor to be considered. The pyrometer must have a wavelength where the glass is somewhat transparent, which means they can only be used for high temperature. Otherwise, the instrument will have measurement errors due to averaging of the glass temperature with the desired product temperature.

MOUNTING

How can I mount the infrared pyrometer?

The pyrometer can be of two types, either fixed-mount or portable. Fixed mount units are generally installed in one location to continuously monitor a given process. They usually operate on line power, and are aimed at a single point. The output from this type of instrument can be a local or remote display, along with an analog output that can be used for another display or control loop.

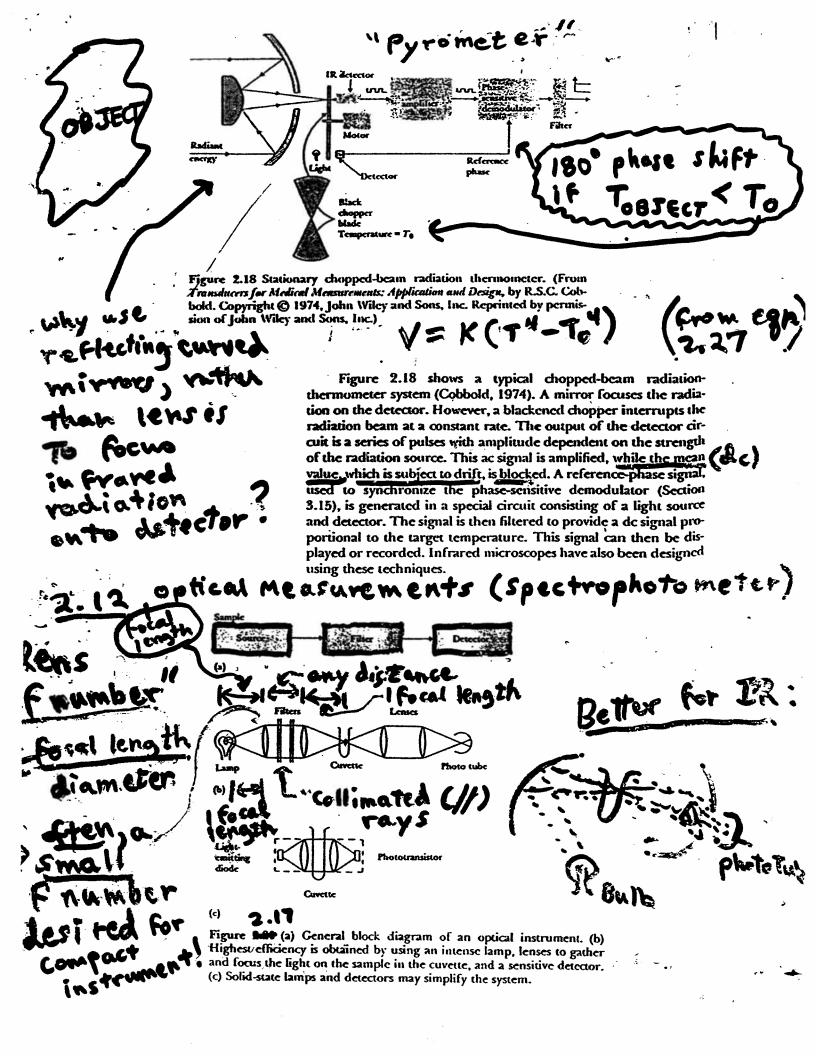
Battery powered, portable infrared "guns" are also available; these units have all the features of the fixed mount devices, usually without the analog output for control purposes. Generally these units are utilized in maintenance, diagnostics, quality control, and spot measurements of critical processes.

RESPONSE TIME

What else should I take into account when selecting and installing my infrared measurement system?

First, the instrument must respond quickly enough to process changes for accurate temperature recording or control. Typical response times for infrared thermometers are in the 0.1 to 1 second range. Next, the unit must be able to function within the environment, at the ambient temperature.

Other considerations include physical mounting limitations, viewing port/window applications (measuring through glass), and the desired signal processing to produce the desired output for further analysis, display or control.



See text p. 453

Example of Optical Measuring Instrument

Blood Oxemeter - Measures fraction of hemogle

(ngb) that is oxygenated (L)

 $\Lambda(\lambda) = WL[a_{\bullet}(\lambda)C_{\bullet} + a_{r}(\lambda)C_{r}]$

Absorbance

Joy [Mois (2)]

Luster (2)

where I

is detector

output

A can

be measured

at different

savelengths

 $A(\lambda)$

weight of hemoglobin per unit volume a_{\bullet} and a_{r} = absorptivities of hgbO and hgb a_{\bullet} and a_{r} = concentrations of hgbO and hgb $(C_{\bullet} + C_{r} = 1.0)$

Figure 2.21(b) shows that a_0 and a_7 are equal at 805 nm, called the isobestic wavelength. If this wavelength is λ_2 , then

WL = A(h) Q \ \ 2 = isobestic = 805nm
2-50 A = Q = Q = Q

(2.49)

where

Therefore $A(\lambda_2) = a_r(\lambda_2)$ $a_r(\lambda_2) = a_r(\lambda_2)$ $a_r(\lambda_1) = a_r(\lambda_2)$ $a_r(\lambda_1) = a_r(\lambda_1) = a$

When absorbance is measured at a second wavelength λ_i , the oxygen saturation is given by

 $C_{\bullet} = x + \frac{\gamma A(\lambda_{\parallel})}{A(\lambda_{\perp})}$ on $\alpha(\lambda_{\parallel})$ $\alpha(\lambda_{\parallel})$

where x and y are constants that depend only on the optical characteristics of blood. In practice λ_1 is chosen to be that wavelength at which the difference between a_r and a_r is a maximum, which occurs at 660 nm. [see Figure 2.21(b)].

Oximeters may use interference filters in the form shown in Figure 2.20(b). They have been built as shown in Figure 2.20(c) using LEDs selected for proper wavelengths. One form uses fiber optics to transmit the wavelengths into and back from the blood within the vessels. Oximeters can be used to noninvasively measure %O₂ saturation by passing light through the pinna of the ear (Merrick and Hayes, 1976). Because of the complications caused by the light-absorbing characteristics of skin pigment and other absorbers, measurements are made at eight wavelengths and computer-processed. The ear is warmed to 41°C to stimulate arterial blood flow.

are known constants

[Mote]

CIFEDS Recombination 200 of hole-electron pairs - emits Photons 220mf energy OF wavelength dictated materials is band Voltage, V Figure 2.22 Forward characteristics for p-n junctions. Ordinary silicon diodes have a band gap of 1.1 eV and are inefficient radiators in the near-infrared. GaAs has a band gap of 1.44 eV and radiates at 900 nm. GaP has a band gap of 2.26 eV and radiates at 700 nm. asers. N2 Sin B2 Snell's Law: Sin & = 四 sin 日 if this value , 02 can be foun Coating ™≈1.2 Air reflection Fiber :. Smalles 6 for which we have internal reflection, Olc found by setting 42 >N. Figure 2.23 Fiber optics. The solid line shows refraction of rays that es-...pe through the wall of the fiber. The dashed line shows total internal redection within a fiber.

Snell's Law

Phase Fronts must match up at boundary between two optical Phase Fronts along boundary must move with same relocity phose fronts med # 2 either side of boundary. Likewise Thany = n2 sin 02 (n, > n)Volny, = (di sine,)/et

Couring (Classified)

Example 2.5 Find the angle θ_1 for the largest cone of light accepted from a medium of refractive index n that is totally reflected within a glass fiber of refractive index n_1 with a coating of refractive index n_2 .

Answer At the interface of medium n and the glass, Snell's law (2.47) yields $n \sin \theta_1 = n_1 \sin \left(\theta_1 - \frac{9\theta^2}{9\theta^2}\right) = n_1 \cos \theta_{10}$. Substituting this result into (2.48) yields $\frac{n_2}{n_1} = \sin \theta_{10} = (1 - \cos^2 \theta_{10})^{1/2} = \left(1 - \frac{n^2}{n_1^2} \sin^2 \theta_3\right)^{1/2}$ Rearranging and taking the square root, $\frac{n}{n_1} \sin \theta_3 = \frac{(n_1^2 - n_2^2)^{1/2}}{n_1}$ yields $\sin \theta_3 = \frac{1}{n} (n_1^2 - n_2^2)^{1/2}$

A 50-cm glass fiber exhibits a transmission exceeding 60% for wavelengths between 400 and 1200 nm. A 50-cm plastic fiber has a transmission exceeding 70% for wavelengths between 500 and 850

Optical Filters (Wavelength Selectors)

A. Newtral Density Filters (Attenuator - Partially Silvered Mirror - Suspended Carbon Particles (Smake - 2 Polaroid Filters (RPFS)

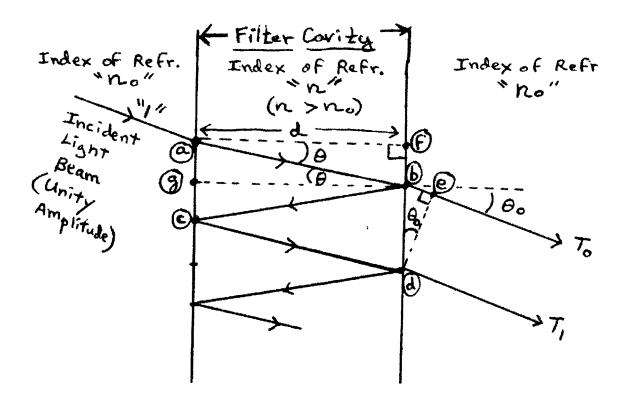
E. Color Filters (RPFS)

- Gelatin Film (IR Kdak 87)

- Dired Glass (Corning 5-56)

. . . p. <

Band Pass Interference Filter (Fabry-Perot Interferometer)



Let us define the wavelength of a light beam of frequency "f" in free space (where the velocity of light in free space is $c = 3x10^8$ m/s) as

$$\lambda = \frac{c}{f}$$

Then we may denote the wavelength of this light beam as it travels through a medium with refractive index n by

$$\lambda_n = \frac{\left(\frac{c}{n}\right)}{f} = \frac{\lambda}{n}$$

At each reflection of a ray from a boundary, the amplitude of the ray's E field wave is multiplied by \sqrt{R} where R is the power reflection coefficient, (0 < R < 1), and the amplitude of the ray's E field wave is inverted, so it undergoes a phase shift of π radians. This is because the characteristic impedance η of a medium with refractive index n is given by $\eta = \frac{377 \cdot \Omega}{n}$ Hence the wave is traveling from a region of higher

refractive index (lower characteristic impedance η) to a region of lower refractive index (higher characteristic impedance η 0), and thus the E-field intensity reflection coefficient at either boundary is give by $(\eta - \eta 0)/(\eta + \eta 0)$, and is therefore <u>negative!</u>

We shall see that for a highly selective bandpass filter, we must require P to be close to

We shall see that for a highly selective bandpass filter, we must require R to be close to unity.

At each transmission, the amplitude of the ray E field is multiplied by \sqrt{T} where T is the power transmission coefficient, (0 < T < 1).

Assuming that there is negligible absorbtion in the media, Energy Conservation requires that

$$T + R = 1$$

Part 1: Finding cavity thickness "d" that selects wavelength λ

From the geometry of the right triangle abf

$$cos(\theta) = \frac{d}{l_{ab}}$$
 (Where lab is the distance from Point a to Point b)

=>
$$l_{ab} = \frac{d}{\cos(\theta)}$$
 This is the distance the light ray travels as it moves from one side of the filter cavity to the other.

Thus the distance traveled by an internally reflected light ray traveling from Point b to Point c to Point d is given by

$$l_{bcd} = 2 \cdot l_{ab} = \frac{2d}{\cos(\theta)}$$
 This represents the distance travelled by a ray of light as it bounces down and back inside the filter cavity.

From the geometry of the right triangle <u>agb</u>

$$\tan(\theta) = \frac{l_{ag}}{d} \implies l_{ag} = d \cdot \tan(\theta)$$

Note
$$l_{bd} = 2 \cdot l_{ag} = 2 \cdot d \cdot tan(\theta)$$

From the geometry of the right triangle <u>bed</u>

$$\sin(\theta_0) = \frac{l_{be}}{l_{bd}} = \frac{l_{be}}{2 \cdot d \cdot \tan(\theta)}$$
 Thus $l_{be} = 2 \cdot d \cdot \tan(\theta) \cdot \sin(\theta_0)$

Assume that the phasor expression for the amplitude of the E field of the incident light beam at the entry <u>point a</u> is $1 \cdot e^{j \cdot 0} \cdot \frac{V}{m}$

Then the phasor expression for the amplitude of the E field of the wave that exits from the cavity at <u>point b</u> (light ray To) must be given by

$$\frac{j \cdot \left(\frac{2 \cdot \pi}{\lambda}\right) \cdot I_{ab}}{To = T \cdot e} \quad j \cdot \left(\frac{2 \cdot \pi \cdot n}{\lambda}\right) \cdot \frac{d}{\cos(\theta)}$$

The phasor expression for the amplitude of the E field of the light ray "To" evaluated at <u>point e</u> (a little further along its path, where it will superimpose with the internally reflected ray T1) is therefore given as

$$To_{e} = T \cdot e^{j \cdot \left(\frac{2 \cdot \pi}{\lambda}\right) \cdot \left(\frac{d \cdot n}{\cos(\theta)} + l_{be} \cdot n_{o}\right)} = T \cdot e^{j \cdot \left(\frac{2 \cdot \pi}{\lambda}\right) \cdot \left(\frac{d \cdot n}{\cos(\theta)} + 2 \cdot d \cdot \tan(\theta) \cdot \sin(\theta_{0}) \cdot n_{o}\right)}$$

Note that the portion of exit ray To that is outside of the filter cavity is traveling in the external medium, which has refractive index n_0 . Thus the additional phase shift term that must be added to correct the phase of To is given by $l_{be} \cdot n_0 = 2 \cdot d \cdot tan(\theta) \cdot sin(\theta_0) \cdot n_0$.

At <u>point d</u>, the phasor expression for the amplitude of the E field of the light ray "T₁" must be

$$T_1 = T \cdot R \cdot e^{3 \cdot \left[j \cdot \left(\frac{2 \cdot \pi \cdot n}{\lambda} \right) \cdot \frac{d}{\cos(\theta)} \right] + j \cdot 2 \cdot \pi}$$

Where the $j \cdot 2 \cdot \pi$ term is due to the two π -radian phase inversions at the two internal reflection points. This term may be removed, since $\exp(j2\pi) = 1$, as shown below:

$$T_{1} = T \cdot R \cdot e^{3 \cdot \left[j \cdot \left(\frac{2 \cdot \pi \cdot n}{\lambda} \right) \cdot \frac{d}{\cos(\theta)} \right] \cdot e^{\left(j \cdot 2 \cdot \pi \right)}} = T \cdot R \cdot e^{3 \cdot \left[j \cdot \left(\frac{2 \cdot \pi \cdot n}{\lambda} \right) \cdot \frac{d}{\cos(\theta)} \right]}$$

Therefore the phase difference between the two beams of light (at point d and point e) is given by

$$\Phi_{d} = \Phi_{T1} - \Phi_{To_{e}} = 3 \cdot \left[\left(\frac{2 \cdot \pi \cdot n}{\lambda} \right) \cdot \frac{d}{\cos(\theta)} \right] - \left(\frac{2 \cdot \pi}{\lambda} \right) \cdot \left(\frac{d \cdot n}{\cos(\theta)} + 2 \cdot d \cdot \tan(\theta) \cdot \sin(\theta_{0}) \cdot n_{o} \right)$$

$$\Phi_{d} = \Phi_{T1} - \Phi_{To_{e}} = 2 \cdot \left(\frac{2 \cdot \pi \cdot n}{\lambda}\right) \cdot \frac{d}{\cos(\theta)} - \frac{2 \cdot \pi}{\lambda} \cdot \left(2 \cdot d \cdot \tan(\theta) \cdot \sin(\theta_{0}) \cdot n_{o}\right)$$

But Snell's law states that

$$n_0 \cdot \sin(\theta_0) = n \cdot \sin(\theta)$$

Thus

$$\Phi_{d} = 2 \cdot \left(\frac{2 \cdot \pi \cdot n}{\lambda}\right) \cdot \frac{d}{\cos(\theta)} - \frac{2 \cdot \pi}{\lambda} \cdot \left(2 \cdot d \cdot \tan(\theta) \cdot \sin(\theta) \cdot n\right)$$
But $\tan(\theta) = \sin(\theta)/\cos(\theta) \implies \oint_{d} = 2 \cdot d \cdot \left(\frac{2 \cdot \pi \cdot n}{\lambda}\right) \left[\frac{1 - \sin^{2} \theta}{\cos \theta}\right] = 2 \cdot \frac{2 \pi \cdot n}{\lambda} \cdot d \cdot \left(\frac{\cos^{2} \theta}{\cos^{2} \theta}\right)$

$$\Phi_{d} = 2 \cdot \frac{2\pi \cdot n}{\lambda} \cdot d \cdot \cos(\theta)$$

For maximum transmittance through the interference filter, we must require that the phase difference between the two emerging beams, To_e and T_1 (and in general, ALL of the emerging beams) must be an integer multiple of 2π radians. Thus we require

$$m \cdot 2 \cdot \pi = 2 \cdot \frac{2 \cdot \pi \cdot n}{\lambda} \cdot d \cdot \cos(\theta)$$
 Where $m = 1, 2, 3, ...$

$$\mathbf{m} \cdot \lambda = 2 \cdot \mathbf{n} \cdot \mathbf{d} \cdot \cos(\theta)$$

Note that "d" is the physical thickness of the filter cavity. The product "nd" is called the optical length, and it is equivalent to the length of an equivalent free-space path.

This result relates the wavelength that is selected to the thickness of the cavity (d), the refractive index of the cavity material (n), and the angle of incidence of the light rays inside the cavity (θ) .

Part 2. Power Transfer Function of the Interference Filter

Recall that the phasor amplitude of the incident ray's E field at entry Point "a" is assumed to be unity. Thus Point a is taken to be our phase reference point, and the phasor amplitude of the m_{th} transmitted beam is given by

$$T_m = T \cdot R^m \cdot e^{j \cdot \Phi_d \cdot m}$$

The phasor amplitude of the E field associated with the total transmitted beam is

$$A_{tot} = T \cdot \sum_{m=0}^{\infty} R^m \cdot e^{j \cdot \Phi_d \cdot m} = T \cdot \sum_{m=0}^{\infty} \left(R \cdot e^{j \cdot \Phi_d} \right)^m$$

Note that we have an infinite sum of a geometric series.

(Aside: Derivation of sum of a semi-infinite geometric series)

Recall how the sum of a geometric series may be written in closed form:

$$\sum_{m=0}^{\infty} a^{m} = 1 + a + a^{2} + a^{3} + \dots$$

Multiplying both sides by "a"

$$a \cdot \sum_{m=0}^{\infty} a^m = a + a^2 + a^3 + a^4 + ...$$

Subtracting the two equations above yields

$$\sum_{m=0}^{\infty} a^m - a \cdot \sum_{m=0}^{\infty} a^m = 1$$

Solving for the summation yields

$$\sum_{m=0}^{\infty} a^m = \frac{1}{1-a}$$

Thus in our case, we may use the semi-infinite geometric series summation formula to express the phasor amplitude of the total transmitted E field as

$$A_{\text{tot}} = T \cdot \frac{1}{1 - R \cdot e^{j \cdot \Phi_d}}$$

The power density transmitted by the light ray is proportional to the squared magnitude of the electric field intensity, just as the power dissipated by a resistor is proportional to the squared magnitude of the voltage across the resistor. Furthermore, recall that a complex variable multiplied by its complex conjugate is the magnitude of that variable, since $(a+jb)*(a-jb) = a^2 + b^2 = |a+jb|^2$.

Therefore, the amplitude of the total transmitted power in the ray is

$$(|A_{tot}|)^2 = A_{tot} \cdot \overline{A_{tot}}$$
 Where $\overline{A_{tot}}$ is the complex conjugate of A_{tot}

$$(|A_{tot}|)^2 = T \cdot \frac{1}{1 - R \cdot e^{j \cdot \Phi_d}} \cdot \left(T \cdot \frac{1}{1 - R \cdot e^{j \cdot \Phi_d}}\right)$$

$$\left(\left|A_{tot}\right|\right)^{2} = \frac{T^{2}}{\left(1 - R \cdot e^{j \cdot \Phi_{d}}\right) \cdot \left(1 - R \cdot e^{-j \cdot \Phi_{d}}\right)}$$

$$\left(\left|A_{tot}\right|\right)^{2} = \frac{T^{2}}{\left(1 - R \cdot \exp\left(-j \cdot \Phi_{d}\right) - R \cdot \exp\left(\frac{i}{2} \cdot \Phi_{d}\right) + R^{2}\right)}$$

$$\left(\left|A_{tot}\right|\right)^{2} = \frac{T^{2}}{\left[1 - R \cdot \left(\cos\left(-\Phi_{d}\right) + j\sin\left(-\Phi_{d}\right)\right) - R \cdot \left(\cos\left(\Phi_{d}\right) + j\sin\left(\Phi_{d}\right)\right) + R^{2}\right]}$$

$$(|A_{tot}|)^2 = \frac{T^2}{\left[1 - R \cdot \cos(\Phi_d) + R \cdot j \sin(\Phi_d) - R \cdot \left(\cos(\Phi_d) + j \cdot \sin(\Phi_d)\right) + R^2\right]}$$
(Due to even symmetry of cosine function and odd symmetry of sin function)

$$(|A_{tot}|)^2 = \frac{(1-R)^2}{(1-2\cdot R\cdot \cos(\Phi_d) + R^2)}$$
 Recall that T+R = 1 => T = 1-R

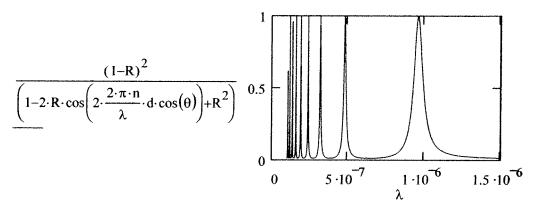
Where
$$\Phi_d = 2 \cdot \frac{2\pi \cdot n}{\lambda} \cdot d \cdot \cos(\theta)$$
 This result may be regarded as the filter's power transfer function, since we assumed the incident light beam was of unit amplitude.

This power transfer function is plotted below for

$$R := 0.80$$
 $n := 1.6$ $d := 0.3 \cdot 10^{-6}$ m $\theta := 0$ radians

m = 1 corresponds to the highest wavelength that can be selected:

for m = 1 =>
$$\lambda := 2 \cdot n \cdot d \cdot \cos(\theta)$$
 $\lambda = 9.6 \times 10^{-7}$ meters
 $\lambda := 0.1 \cdot 10^{-6}, 0.101 \cdot 10^{-6} ... 1.5 \cdot 10^{-6}$



The higher the reflectivity, the sharper the passband. Here R has been increaseed from 0.80 to 0.95

$$R := 0.95$$
 $n := 1.6$ $d := 0.3 \cdot 10^{-6}$ m $\theta := 0$ radians

$$\lambda := 0.1 \cdot 10^{-6}, 0.101 \cdot 10^{-6} ... 1.5 \cdot 10^{-6}$$

$$\frac{(1-R)^2}{\left(1-2\cdot R\cdot \cos\left(2\cdot \frac{2\cdot \pi\cdot n}{\lambda}\cdot d\cdot \cos(\theta)\right)+R^2\right)} \quad 0.5$$

$$0$$

$$0$$

$$0$$

$$0$$

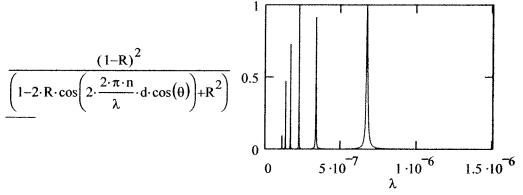
$$1\cdot 10^{-6}$$

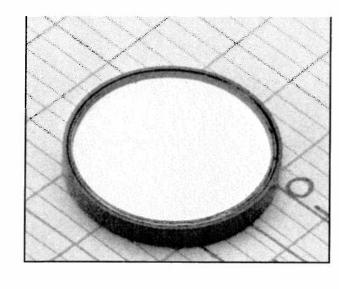
$$1.5\cdot 10^{-6}$$

The angle of incidence of light on the interference filter shifts its passband and therefore permits an interference filter to be tunable, to some extent. Here θ has been changed from 0 radians to 0.80 radians:

$$R := 0.95 \quad n := 1.6 \quad d := 0.3 \cdot 10^{-6} \quad m \quad \theta := 0.80$$
for m = 1 \qquad \tau := 2 \cdot n \cdot d \cdot \cos(\theta) \quad \tau = 6.688 \times 10^{-7} \quad \text{meters}

$$\lambda := 0.1 \cdot 10^{-6}, 0.101 \cdot 10^{-6} ... 1.5 \cdot 10^{-6}$$





Interference Filters

INTERFERENCE FILTERS: WHAT THEY ARE AND HOW TO USE THEM

Interference filter applications are extremely diverse, including disease diagnosis, spectral radiometry, calorimetry, and color separation in television cameras. Used with even the least expensive broadband photometers or radiometers, Melles Griot interference filters enable rapid and accurate measurement of spectral distributions. This combination has an enormous throughput advantage since the collecting area of filters is very large compared to instrumental slits. Additionally, interference filters make viewing and near-instantaneous recording of very spectrally selective images possible. Spatial and spectral scanning instruments can provide similar images but take much longer.

Narrowband interference filters permit isolation of wavelength intervals a few nanometers or less in width, without dispersion elements such as prisms or gratings. For example, a single line in the emission spectrum of a flame can be monitored without confusion from other nearby lines, or the signal from a laser communications transmitter can be received without interference from a brightly sunlit landscape. Colored glass and gelatin filters are incapable of such discrimination.

Interference filters are multilayer thin-film devices. While many interference filters can be correctly described as "all dielectric" in construction, metallic layers are often present in auxiliary blocking structures. Broadband interference filters almost always contain a metallic layer (in their spacer, not in their stacks). Interference filters come in two basic types, edge filters and bandpass filters, which transmit a desired wavelength interval while simultaneously rejecting both longer and shorter wavelengths.

FABRY-PEROT INTERFEROMETER

Narrowband interference filters (bandpass filters) operate with the same principles as the Fabry-Perot interferometer. In fact, they can be considered Fabry-Perot interferometers since they usually operate in the first order.

The Fabry-Perot is a simple interferometer, which relies on the interference of multiple reflected beams. The figure at the right shows a schematic Fabry-Perot cavity. Incident light undergoes multiple reflections between coated surfaces which define the cavity. Each transmitted wavefront has undergone an even number of reflections $(0, 2, 4, \ldots)$. Whenever there is no phase difference between emerging wavefronts, interference between these wavefronts

produces a transmission maximum. This occurs when the optical path difference is an integral number of whole wavelengths, i.e., when

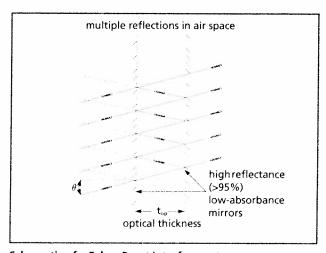
$$m\lambda = 2t_{op}\cos\theta$$

where m is an integer, often termed the order, t_{op} is the optical thickness, and θ is the angle of incidence. At other wavelengths, destructive interference of transmitted wavefronts reduces transmitted intensity toward zero (i.e., most, or all, of the light is reflected back toward the source).

Transmission peaks can be made very sharp by increasing the reflectivity of the mirror surfaces. A simple Fabry-Perot interferometer transmission curve is shown on the next page. The ratio of successive peak separation to full width at half-maximum (FWHM) transmission peak is termed finesse. High reflectance results in high finesse, or high resolution.

In most Fabry-Perot interferometers, air is the medium between high reflectors; therefore, the optical thickness, $t_{\rm op}$, is essentially equal to d, the physical thickness. The air gap may vary from a fraction of a millimeter to several centimeters.

The Fabry-Perot is a useful spectroscopic tool. It provided much of the early motivation to develop quality thin films for the high-reflectance mirrors needed for high finesse. Fabry-Perot interferometers can be constructed from purely metallic coatings, but high absorption losses limit performance.

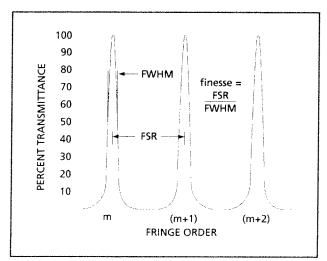


Schematic of a Fabry-Perot interferometer

BANDPASS FILTER DESIGN

The simplest bandpass filter is a very thin Fabry-Perot interferometer. The air gap is replaced by a thin layer of dielectric material with a half-wave optical thickness (optimized at the wavelength of the desired transmission peak). The high reflectors are normal quarterwave stacks with a broadband reflectance peaking at the design wavelength.

The entire assembly of two quarterwave stacks, separated by a half-wave spacer, is applied to a single surface in one continuous vacuum deposition run. By analogy with interferometers, the simplest bandpass interference filters are sometimes called cavities. Two or more such filters can be deposited on top of the other, separated by an absentee layer, as shown in the figure on the next page, to form a multiple-cavity filter. The resulting overall transmittance passband is given approximately by the product of the passbands of individual cavities. The advantages of multiple-cavity filters are steeper band slopes, improved near-band rejection, and "square" (not Gaussian or Lorentzian) passband peaks. This last result, especially desirable in intermediate bandwidth filters, is achieved in part by reducing stack reflectance, which broadens individual cavity passbands.



Transmission pattern showing the free spectral range (FSR) of a simple Fabry-Perot interferometer.

The figure on the next page shows a cross-sectional view of a typical two-cavity interference filter. The effect of the number of cavities on final passband shape, normalized to passband FWHM, is shown on page 13.28. The figure on page 13.27 shows a detailed structure of the all-dielectric multilayer bandpass filter film. H symbolizes precisely a quarter-wavelength optical thickness layer of a high-index material (typically zinc sulfide), while L symbolizes a precisely quarter-wavelength optical thickness layer of a low-index material (typically cryolite, Na₃AIF₆). The spacer is a layer of high index material of half-wavelength thickness, and the absentee, or coupling, layer is a layer of low-index material of half-wavelength thickness. Wavelength refers to the wavelength of peak transmittance. Layers are formed by vacuum deposition. The aluminum rings protect the edges, and epoxy cement protects the films from moisture and laminates the bandpass and blocker sections together. Depending on the center wavelength and FWHM, other filters may have many more layers than those illustrated.

ADDITIONAL BLOCKING

Close to the passband, and on the long wavelength side, multilayer blocking structures (usually metal dielectric hybrid filters) are used in Melles Griot passband filters to limit transmittance to 0.01%. More stringent blocking is possible, but this increases filter cost and compromises maximum transmission. Colored glass is often used to suppress transmission on the short wavelength side of the passband.

TABLE OF NORMALIZED PASSBAND SHAPE

The graph on page 13.28 showing the that change in performance with an increasing number of cavities is qualitatively useful, but the bandwidth table on page 13.28 gives quantitative data. This table applies to zinc sulfide (ZnS)/cryolite (Na₃AlF₆) interference filters of any FWHM.

Although the table is strictly applicable from 400 nm to 1100 nm, Melles Griot ultraviolet filters, which are of different composition, have very similar characteristics. The table shows functional dependence of normalized passband shape on the number of cavities used in filter construction, with FWHM arbitrary but held fixed. Because transmittance is normalized to peak value, the table is applicable to blocked and unblocked filters. To apply the table to a specific filter, simply multiply by peak transmittance. Both

minimum and maximum full bandwidths are shown at various normalized transmittance levels. The difference between minimum and maximum full bandwidths allows for spacer material choice and filter-to-filter variation. Normal incidence is assumed. Beyond the spectral range displayed here, our filters of two, three, and four cavity construction are supplied with blocking structures that limit absolute transmittance to less than 10^{-4} .

WAVELENGTH DEPENDENCE ON ANGLE OF INCIDENCE

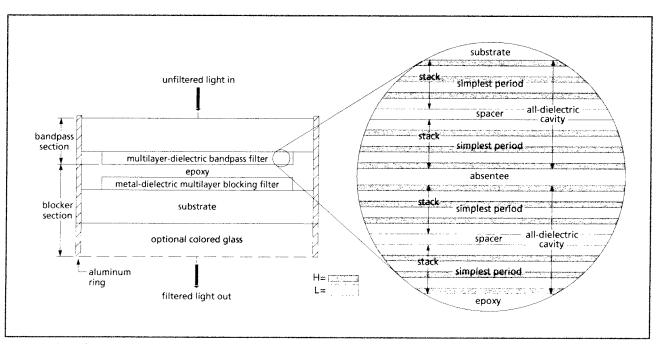
A common characteristic of single and multilayer dielectric coatings and interference filters is that transmittance and reflectance spectra shift to shorter wavelengths as they are tilted from normal to oblique incidence. This applies to both edge and bandpass filters. As tilt is increased in filters constructed with metallic layers, the transmittance peak splits into two orthogonally polarized peaks which shift to shorter wavelengths at different rates. Melles Griot narrowband filters are made with all-dielectric multilayers to prevent this transmittance split from occurring.

The shift to shorter wavelengths at oblique incidence is very useful in tuning bandpass filters from one wavelength to another, or adjusting the half-power point wavelengths of edge filters, in collimated light. If the peak transmittance wavelength is slightly too long, tilt the filter. This ability to shift wavelength enhances the usefulness to interference filter sets. Each filter in variable bandpass sets can be angle tuned down to the normal incidence transmission wavelength of the next filter in the set.

Wavelengths of transmittance peaks or cavity resonances for Fabry-Perot interferometers and bandpass interference filters are approximately governed, for observers within the cavity or spacer, by the equation

$$2n_e t \cos\theta = m\lambda$$

where n_e is spacer refractive index, t is the spacer thickness, θ is the internal angle of incidence (measured within the cavity or spacer), m is the order number of interference (a nonzero positive integer), and λ is the wavelength of a particular resonance transmittance

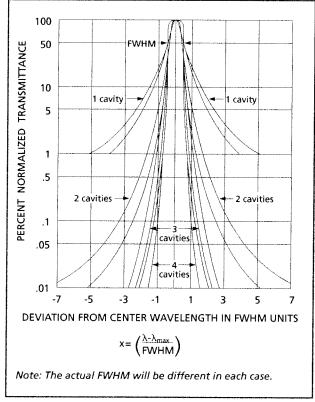


Cross section of a typical two-cavity pass interference filter

peak. This equation is often called the monolayer approximation. The formula can be satisfied simultaneously for many different order number and wavelength combinations. Corresponding to each such combination there is, in principle, a different resonance transmittance peak for an unblocked filter. For an all-dielectric filter there are, between cavity resonance transmittance peaks, additional broader peaks that correspond to the wavelengths at which the dielectric stacks are ineffective as resonant reflectors. Only a single resonance transmittance peak is selected for use and allowed to appear in the output spectrum of a complete (blocked) interference filter. Blocking techniques are highly effective.

Bandwidth at Various Normalized Transmitters

		Multiplier for	
	Normalized Transmittance	Full Bandwidth at Indicated Transmittance	
Number			
of	Level	Minimum	Maximum
Cavities	(% of peak)	(in FWHM units)	
1	90	0.3	0.35
	10	2.5	3.0
	1	8.0	10.0
2	90	0.5	0.6
	10	1.6	2.0
	1	2.8	3.5
	0.1	5.5	6.3
	0.01	10.0	15.0
3	90	0.7	0.8
	10	1.2	1.5
	1	1.9	2.2
	0.1	2.9	3.2
	0.01	4.9	5.4
4	90	0.85	0.9
	10	1.1	1.25
	1	1.5	1.65
	0.1	2.0	2.25
	0.01	3.5	4.25



Normalized log scale transmittance curves for ZnS/Na₃AlF₆ bandpass interference filters of 10 nm FWHM as functions of the number of cavities used in filter construction

FWHM EXAMPLE

A three-cavity filter at the 1% normalized transmittance level (1% of peak) would have a nominal full bandwidth (full width at 1% of maximum) between the limits of 1.9 and 2.2. If the FWHM were 5.0 nm, the FW1%M would be between 9.5 and 11.0 nm.

In terms of the external angle of incidence, ϕ , it can be shown that the wavelength of peak transmittance at small angles of incidence is given by

$$\lambda = \lambda_{\text{max}} \sqrt{1 - \left(\frac{n_o}{n_e}\right)^2 \sin^2 \phi}$$

where n_0 is the external medium refractive index ($n_0 = 1.0$ in air) and n_e is the spacer effective refractive index. The difference λ_{max} $-\lambda$ is the *angle shift*. The spacer effective index is dependent on wavelength, film material, and order number because of multilayer effects. The effective index and actual refractive index of spacer material is not equivalent, although the same symbol n_a is used for both. By curve-fitting the second formula above (from which t is absent) to measured angle shifts at small angles, the effective index and angle at which blocker displacement of the peak becomes significant can, in principle, be found. In the absence of actual measurements, the formula probably should not be trusted much beyond five or ten degrees. With suitable interpretation, the formula can be applied to prominent landmarks in transmittance and reflectance spectra of edge filters, multi- and single-layer coatings, and all interference filters.

In many applications, angle shifts can be safely ignored. Advanced radiometer designs are necessary only when simultaneously wide fields and narrow bandwidths are required. For example, if the desired monochromatic signal is to be at least 90% of T_{peak} throughout the field and the filter has a narrow 1.0-nm FWHM, angular radius is only about 2.5 degrees.

Most Melles Griot filters use a high-index spacer (usually zinc sulfide) to minimize angle shift. Some use low-index spacers (usually cryolite) to achieve higher transmittance or narrower bandwidths.

CORRECT INTERFERENCE FILTER ORIENTATION

A good rule of thumb, especially important if there is risk of overheating or solarization, is that interference filters should always be oriented with the shiniest (metallic) and most nearly colorless side toward the source in the radiant flux. This orientation will minimize thermal load on the absorbing-glass blocking components. Reversing filter orientation will have no effect on filter transmittance near or within the passband.

TEMPERATURE EFFECTS, LIMITS, AND THERMAL SHOCK

The transmittance spectrum of an interference filter is slightly temperature dependent. As temperature increases, all layer thicknesses increase. At the same time, all layer indices change. These effects combine in such a way that the transmittance spectrum shifts slightly to longer wavelengths with increasing temperature. Thermal coefficient is a function of wavelength, as shown in the following table.

Melles Griot interference filters are designed for use at 20°C. Unless bandpass filters with extremely narrow FWHMs are used at very different temperatures, the transmittance shifts indicated in the table are negligible. Our standard interference filters can be used at temperatures down to -50°C. Thermal contraction will result in permanent filter damage below this temperature. Hightemperature limits depend on filter design: 70°C is a safe and conservative limit for all filters. Some of our standard filters can accommodate temperatures up to 125°C. As a general rule, it is unwise to subject interference filters to thermal shock, especially as the lower limit of -50° C is approached. Temperature change rates should not exceed 5°C per minute.

Temperature Dependence of Peak Transmittance

Wavelength (nm)	Temperature Coefficient of Shift (nm per °C)	
400	0.016	
476	0.019	
508	0.020	
530	0.021	
557	0.021	
608	0.023	
630	0.023	
643	0.024	
710	0.026	
820	0.027	

Radiation Sources: Incandescent, Arc Discharge, LED, Laser

1. Incandescent Light Bulb

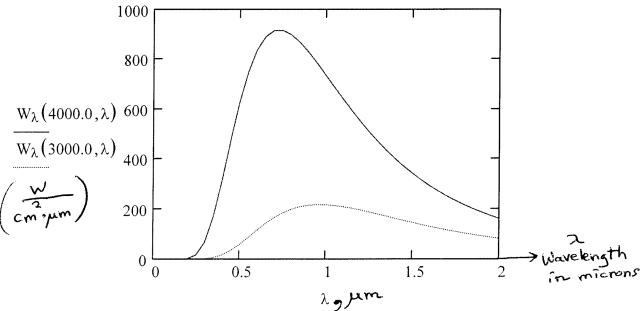
Tungsten Lamp - Tungsten (W) has high melting point and an emissivity of

$$\varepsilon := 0.7$$

$$W_{\lambda}(\text{Temp}, \lambda) := \frac{\epsilon \cdot 3.74 \cdot 10^4}{\lambda^5 \cdot \left(exp \left(\frac{1.44 \cdot 10^4}{\lambda \cdot \text{Temp}} \right) - 1 \right)} \qquad \frac{W}{cm^2 \cdot \mu m}$$

Note that as filament temperature increases from 2000K to 3000K, the spectral peak shifts toward shorter wavelengths.

$$\lambda := 0.2, 0.25...2$$



Also note that much of radiation is in the infrared ($\lambda > 0.7 \mu m$)

Tungsten lamps an inefficient source of light, and only approximately "white" (perfectly white => equal intensities at ALL visible wavelengths.)

Biggest Problem: Tungsten filament evaporates over time...depositing Tungsten on glass bulb, making bulb grow dark after several hundered hours of operation, so the bulb brightness gradually dims over time.

Solution: "Halogen incandescent bulbs" -- Iodine or Bromine (halogen) gas is added to the inert (Nitrogen) gas inside the bulb (Nitrogen is there to replace oxygen to keep W filament from oxidizing). The halogen gas serves as a "custodian" to evaporate the tungsten that is deposited on the glass wall and deposit it back on the filament!

2. Arc Discharge Light Bulbs

A. Fluorescent Lamp

Bulb is filled with low-pressure Ar-Hg mixture. Electrons are accelerated and collide with the gas atoms, which are raised to an excited level. As atom undergoes a transition from a higher energy level to a lower one, the atom emits a quantum of energy (photon). The photon energy E corresponds to the *band gap* of the material, and is related to the frequency of the emitted electromagnetic radiation (v) through the Einstein relationship:

$$E = hv$$

Where
$$h = Planck's constant$$
 $h := 6.63 \cdot 10^{-34}$ J·s

But we also know that frequency of a radiated periodic electromagnetic wave (v) is related to its wavelength (λ) , which is the distance the wave travels before it repeats, through the speed at which the wave travels (c).

$$\lambda = \frac{c}{v}$$
 A dimensional analysis reveals
$$\frac{\left(\frac{m}{s}\right)}{\left(\frac{1}{s}\right)} = m$$

Thus, Einstein's relationship may be written as

$$E = \frac{h \cdot c}{\lambda}$$

Solving for wavelength of the emitted photon as a function of the material bandgap energy, E.

$$\lambda = \frac{h \cdot c}{E}$$
 where the units of "h" above indicate that c is in meters/second and E is in Joules

But often, band gap energy is expressed in a more convenient (much smaller) unit of energy called the "electron volt" (ev) where 1 ev = energy imparted to a single electron when accelerated through a 1 volt potential. From the definition of voltage, 1 V = 1 J / 1 C, or the energy of a Coulomb of charge is raised by 1 J when it is accelerated through 1 V. Thus 1 electron, whose charge is $-1.602*10^-19 C$ would be raised by $(1 J/C)(1.602*10^-19 C) = 1.602*10^-19 J$ when accelerated through 1 V. Thus

$$1 \text{ ev} = 1.602*10^{-19} \text{ J}$$

So if E is in electron volt units, call it E_{ev} , the above formula becomes

$$\lambda = \frac{h \cdot c}{E} = \frac{6.63 \cdot 10^{-34} \cdot J \cdot s \cdot \left(3 \cdot 10^{8} \cdot \frac{m}{s}\right)}{E_{ev} \cdot 1.602 \cdot 10^{-19} \cdot \frac{J}{ev}}$$

$$\lambda = \frac{1240 \cdot 10^{-9}}{E_{ev}} \quad \text{meters}$$

$$\lambda = \frac{1240}{E_{ev}}$$
 nanometers

This final result is a very handy conversion between wavelenth and bandgap!

The main bandgap of mercury is about 5 ev, thus

$$\lambda_{\text{Hg}} := \frac{1240}{5} \qquad \qquad \lambda_{\text{Hg}} = 248 \qquad \text{nm}$$

But this is in the invisible ultraviolet (UV) range. So a white powdered mix of UV-fluorescent dyes that fluoresce (glow under UV stimulation) at several different wavelengths spaced over the visible range of wavelengths (from 0.4 - 0.7 μm) is used to convert the invisible UV radiation into nearly perfectly white light (equal intensity visible radiation over the entire band of visible wavelengths). Fluorescent bulbs are very efficient compared to incandescent bulbs, since little energy is radiated outside of the visible band of wavelengths.

Other low-pressure lamps: Neon and Sodium Vapor (lower bandgaps of these materials permit direct generation of visible light)

High-pressure arc-discharge lamps: Xenon flash tube, Carbon Arc

3. Light Emitting Diodes

As minority charge carriers diffuse across the forward-biased junction of a diode, they become majority carriers, and soon recombine. As they recombine they change from the conducting to the bound state, and lose a photon of energy equal to the bandgap of the material

For GaAs LEDs, the bandgap is 1.44 ev, thus

$$\lambda_{GaAs} := \frac{1240}{1.44}$$
 $\lambda_{GaAs} = 861.111$ nm
(Near Infrared Region)

For GaAsP LEDs, the bandgap is 2.26 ev

$$\lambda_{\text{GaAsP}} := \frac{1240}{2.26}$$
 $\lambda_{\text{GaAsP}} = 548.673$ nm (Green)

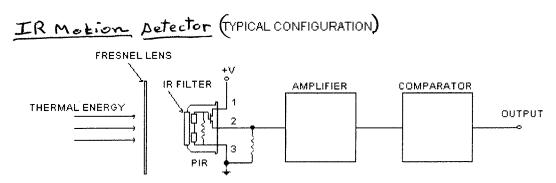
2.16 Radiation Detectors

A. Thermal Detectors

Thermal detectors respond equally well to a wide range of wavelengths. They include Radiation Thermopiles and Pyroelectric (PIR) Sensors. Radiation thermopile detectors were already discussed..

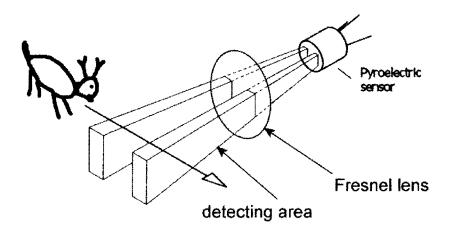
PIR sensors are common and inexpensive IR thermal detectors. They are made from a crystal that exhibits pyroelectric properties. A pyroelectric sensor is made of a crystalline material that generates a surface electric charge when exposed to heat in the form of infrared radiation. When the amount of radiation striking the crystal changes, the amount of charge also changes and can then be measured as a small voltage across electrodes placed on opposite sides of the crystal, using a sensitive field effect transistor (FET) as an amplifying device built into the sensor. The sensor elements are sensitive to radiation over a wide range so a filter window is added to the TO5 package to limit detectable radiation to the 8 to 14mm range which is most sensitive to human body radiation.

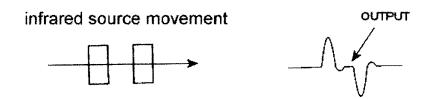
Typically, the FET source terminal pin 2 connects through a pulldown resistor of about 100 K to ground and feeds into a two stage amplifier having signal conditioning circuits. The amplifier is typically bandwidth limited to below 10 Hz to reject high frequency noise and is followed by a window comparator that responds to both the positive and negative transitions of the sensor output signal. A well filtered power source of from 3 to 15 volts should be connected to the FET drain terminal pin 1.



The PIR325 sensor has two sensing elements connected in a voltage bucking configuration. This arrangement cancels signals caused by vibration, temperature changes

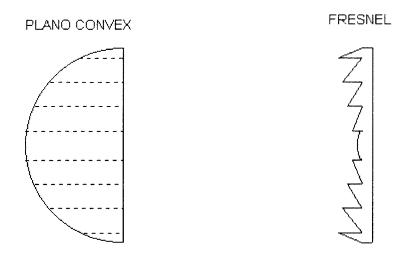
and sunlight. A body passing in front of the sensor will activate first one and then the other element whereas other sources will affect both elements simultaneously and be cancelled. The radiation source must pass across the sensor in a horizontal direction when sensor pins 1 and 2 are on a horizontal plane so that the elements are sequentially exposed to the IR source. A focusing device is usually used in front of the sensor





Fresnel Lens

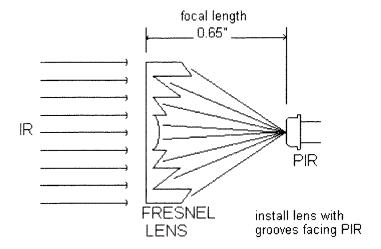
A Fresnel lens (pronounced Frennel) is a Plano Convex lens that has been collapsed on itself to form a flat lens that retains its optical characteristics but is much smaller in thickness and therefore has less absorption losses.

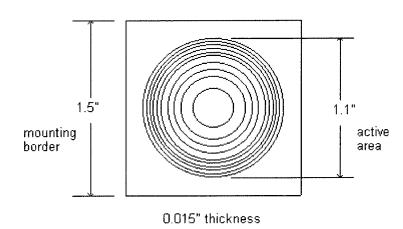


The FL65 Fresnel lens is made of an infrared transmitting material that has an IR transmission range of 8 to 14um which is most sensitive to human body radiation. It is designed to have its grooves facing the IR sensing element so that a smooth surface is presented to the subject side of the lens which is usually the outside of an enclosure that houses the sensor.

The lens element is round with a diameter of 1 inch and has a flange that is 1.5 inches square. This flange is used for mounting the lens in a suitable frame or enclosure. Mounting can best and most easily be done with strips of Scotch tape. Silicone rubber can also be used if it overlaps the edges to form a captive mount. There is no known adhesive that will bond to the lens material.

The FL65 has a focal length of 0.65 inches from the lens to the sensing element. It has been determined by experiment to have a field of view of approximately 10 degrees when used with a PIR325 Pyroelectric sensor.





Optimum transmittance in the 8 to 14 um region

B. Quantum (Photon) Detectors

(These detectors absorb energy from photons and use it to release electrons from the detector material.)

- a. Eye
- b. Photographic Film
- c. Photoconductive Cell (CdS cell)
- d. Photodiode
- e. Phototransistor
- f. Photomultiplier Tube

C. Photomultiplier Tube

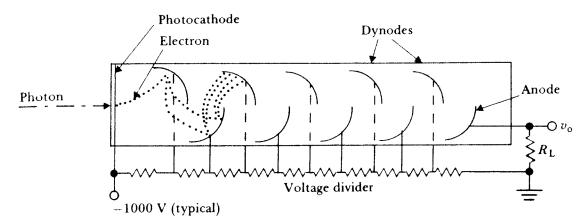


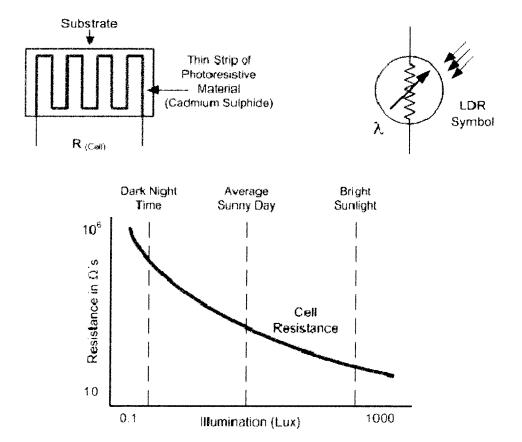
Figure 2.21 Photomultiplier An incoming photon strikes the photocathode and liberates an electron. This electron is accelerated toward the first dynode, which is 100 V more positive than the cathode. The impact liberates several electrons by secondary emission. They are accelerated toward the second dynode, which is 100 V more positive than the first dynode. This electron multiplication continues until it reaches the anode, where currents of about $1 \mu A$ flow through R_L .

The photocathode and dynodes are made of alkali metal with a work function of about 1 eV. Thus a photon energy of 1 eV or greater (about 1.2 micron wavelength or shorter) is needed to knock an electron loose from the photocathode. See the "S4" photocathode response in Text Fig. 2.18(c). The electron is accelerated toward first dynode, which is at about 100 V higher potential. There it knocks loose several additional electrons, which are attracted to the next dynode, where each of these electrons knocks loose several more electrons. This electron multiplying process is repeated from dynode to dynode, resulting

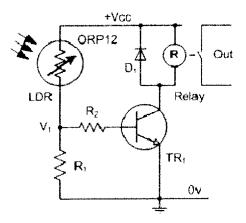
in a relatively large current collected at the anode when just a few photons of light hit the photocathode!

Since thermal activity can knock loose additional electrons from the dynodes and the photocathodes, the photomultiplier may be too noisy at room temperature. Thus in low-light applications, the entire tube may have to be cooled down to liquid nitrogen temperatures. Under this condition, the photomultiplier tube is capable of detecting a single photon of light. By comparison, the human eye can detect as few as 6 photons of light.

D. Photoconductive Cell (CdS, PbS cells). Light-dependent resistor, sometimes called LDR. Bulk doped semiconductor material deposited on a ceramic substrate in a zigzag fashion. Photons with energy in excess of the bandgap of the material will free up the loosely bound charge carriers at the impurity sites in much the same way that in a thermal agitation will knock loose charge carriers in a thermistor. As with the thermistor, the photoconductive cell is rather sensitive, but highly nonlinear, since as light intensity increases, fewer charge carriers are left to be knocked loose.



CdS Cell used in a dawn-to-dusk light relay control security light switching circuit:



E. Photodiode – reverse-biased diode with light-accessible junction.

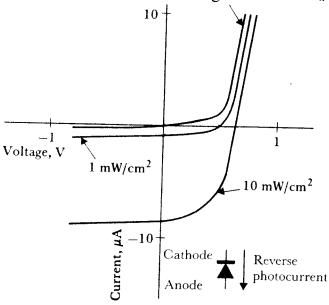
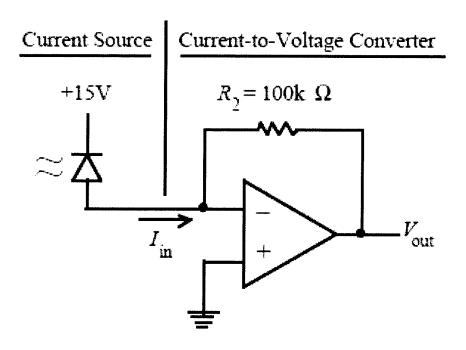


Figure 2.22 Voltage-current characteristics of irradiated silicon p-n junction. For 0 irradiance, both foward and reverse characteristics are normal. For 1 mW/cm², open-circuit voltage is 500 mV and short-circuit current is 0.8 μ A. For 10 mW/cm², open-circuit voltage is 600 mV and short-circuit current is 8 μ A.

With no light hitting PN junction, only a very small reverse saturation current "Is" (now called "dark current") crosses the junction from cathode to anode. The value of Is is very small...typically around 1 nA. Recall that, in a non-illuminated reverse-biased PN junction, this small reverse-bias current is due to the infrequent generation of electron-hole pairs in the depletion region of the reverse-biased junction.

However, when light hits the junction, many more electron-hole pairs are generated in the depletion region, and the reverse-bias diode current increases by several orders of magnitude (into the microampere range). If the light intensity doubles, so does this current. Thus the photodiode is a highly linear photodetector. It also has a relatively fast (on the order of 100 ns) response time. However, it is not very sensitive, and the small reverse currents that flow under optical stimulation are typically in the microampere range. Therefore, the photodiode is often used with an operational amplifier "current-to-voltage" converting circuit, such as the one shown below, where Vout = $\lim_{t \to \infty} 10^{5}$.



Note from the I-V curve above that when the diode junction is dark, we the I-V curve looks like that of any other diode, and is given by the familiar Boltzmann diode equation: I = Is(exp(V/Vt)-1).

However, when the junction is illuminated, the entire diode I-V curve is shifted down vertically. This observation will be important in the next section that deals with photovoltaic cells.

F. Photovoltaic Cell

The photodiode I-V operation discussed above corresponds to the bottom left (I < 0, V < 0) quadrant of the 10 mW/cm^2 illuminated Si photodiode I-V curve shown in the previous

section. However, the bottom right quadrant (I < 0, V > 0) of this same curve is called the photovoltaic quadrant. This quadrant shows that the open circuit voltage (V axis intercept) of the I-V curve for the illuminated S i photodiode varies between V and V depending upon the level of illumination. Also, the short-circuit current (V axis intercept) varies between V and V and V depending on illumination level.

When a load resistor R_L is placed across the 10 mW/cm^2 illuminated diode, we must require that the given photodiode I-V curve holds as well as the external Ohm's Law requirement: $V = -IR_L$. Solving for I, we get the load line equation $I = -V/R_L$. Likening this equation to the equation of a straight line, I = mV + b, we see that this line has a slope of $m = -1/R_L$ and that this load line also passes through the origin, since since the I-intercept, b = 0. The intersection of this load line and the I-V curve yields the diode operating voltage and current. The value of R_L that extracts maximum power from the illuminated photodiode (now called a photovoltaic cell) can be found by finding the intersection point that yields maximum power P = I * V. By drawing a line through this intersection point and the origin, and finding the slope of this line $(= -1/R_L)$, we can find the value of R_L that extracts maximum power.

G. Phototransistor (PT)

The phototransistor has its base region optically accessible, and thus incoming light stimulates base current. Phototransistors may be operated in either common-emitter or common-collector configurations, as shown in the diagrams below. In the common-emitter circuit, the presence of light means that the output voltage moves to a lower value, whereas in the common-collector circuit, the presence of light means that the output voltage changes to a higher value.

In both circuits the phototransistor can be used in two modes, an active "analog" output voltage mode and a switch mode.

Operating in the active mode means that the phototransistor generates an analog (continuously varying) voltage that is roughly proportional to the light received by the component up to a certain light level. When the amount of light surpasses that level, the phototransistor becomes saturated and the output will not increase even as the light level increases. This mode is useful in applications where it is desired to detect two levels of inputs for comparison.

Operating in the switch mode means that the phototransistor will either be "off" (cut-off) or "on" (saturated) in response to the light. This mode is useful when a digital output is required for object detection or encoder sensing.

By adjusting the load resistor in the amplifier circuit one can set the mode of operation. The correct value for the resistor can be determined by the following equations:

Active Mode: Vcc > RL*Icc Switch Mode: Vcc < RL*Icc

Typically a resistor value of $5k\ \Omega$ or higher is adequate to operate the phototransistor in the switch mode. The high level output voltage in the switching mode should equal the supply voltage. The low level output voltage in the switching mode should be less than 0.8 Volts.

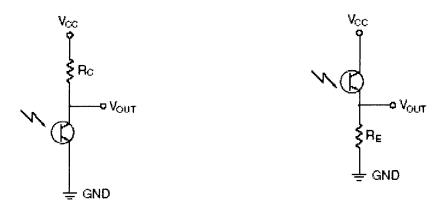


Figure 1. Common-Emitter Amplifler

Figure 2. Common-Collector Amplifier

The PT is approximately 100 times more sensitive than a photodiode circuit, since the optically stimulated current is amplified by the current gain of the transistor (beta), which typically is around 100. However, because beta changes with collector current, the PT is less linear than the photodiode. The PT does not respond as fast to light changes as the photodiode, since there is base charge removal time needed before the PT can be switched off, as with any saturating BJT inverter circuit.

H. PhotoDarlington Transistor

The PhotoDarlington transistor is merely a cascaded Darlington connection of a PT and a normal BJT. The emitter current from the PT is connected to the base of a normal BJT, and the collectors of the two devices are tied together. Thus the optically stimulated current generated by the PT is further amplified by the normal BJT, making the PhotoDarlington transistor about 100 times more sensitive than the regular PT.

The entire circuit is usually housed in one package. The PhotoDarlington may be connected in either the common emitter or the common collector configuration. The common collector configuration is depicted in the circuit shown below.

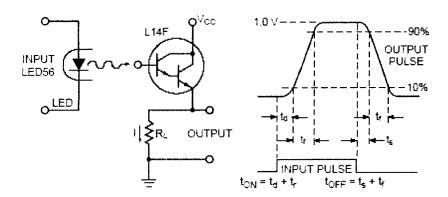


Figure 5. Test Circuit and Voltage Waveforms

The PhotoDarlington transistor is even LESS linear than the PT, since two BJTs are involved, each with a beta that depends on collector current. Also, the PT is even slower to respond to light changes than the PT, since there are two BJTs from which excess base charge must be removed. Often a pull-down resistor is connected between the emitter of the phototransistor and ground in order to help drain the excess base charge from the normal BJT, though this decreases sensitivity, it does speed up the response time of the PhotoDarlington.