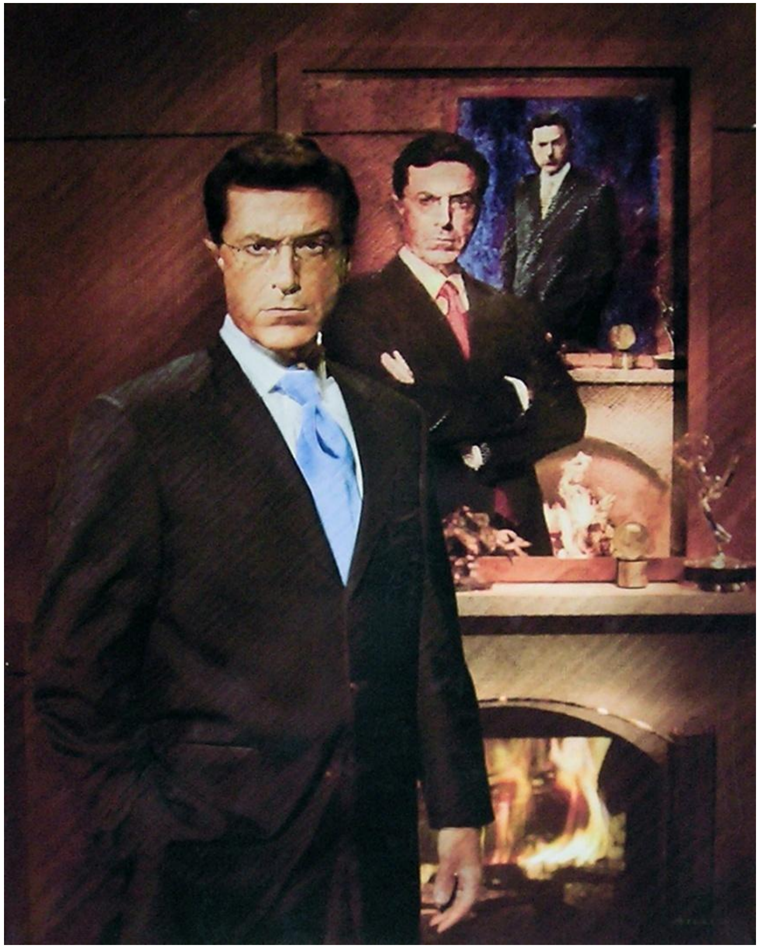

CSSE 220 Day 22

Recursion, Efficiency, and the Time-Space Trade Off; Selection Sort and Big-Oh

Checkout *Recursion2* project from SVN

Questions?


Bizarro
I have this recurring dream that I'm lying here telling you about a recurring dream about lying here telling you about a recurring dream about...
zzzz
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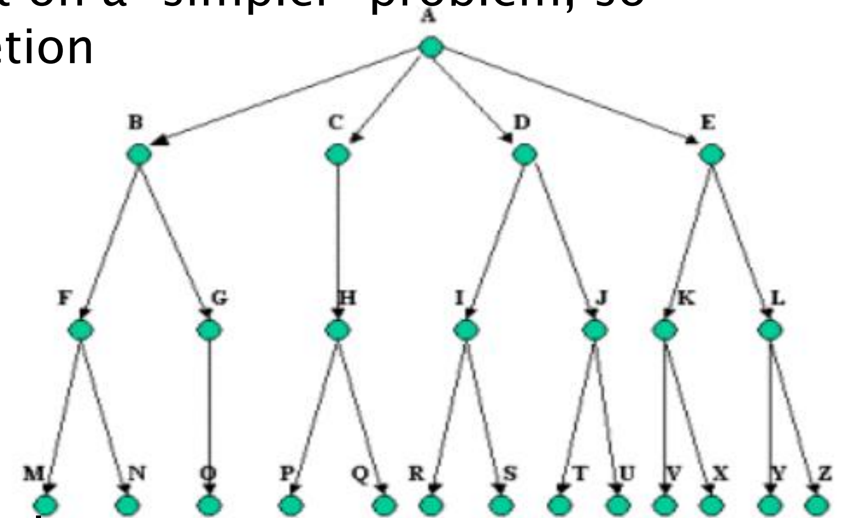
BIZARRO.COM

Recursion

- What is a *recursive* method?
- Answer: *A method that calls itself* but on a “simpler” problem, so that it makes progress toward completion



- When to use recursive methods?
  - Implementing a recursive definition

    $$n! = n \times (n-1)!$$
  - Implementing methods on a recursive data structure, e.g.:

    Size of tree to the right is the sum of sizes of subtrees B, C, D, E, plus 1
  - Any situation where parts of the whole look like mini versions of the whole
    - Folders within folders on computers
    - Trees
- Pros: easy to implement, easy to understand code, easy to prove code correct
- Cons: takes more space than equivalent iteration (because of function calls)

Key Rules to Using Recursion

- Always have a base case that doesn't recurse

- Make sure recursive case always makes progress, by solving a smaller problem

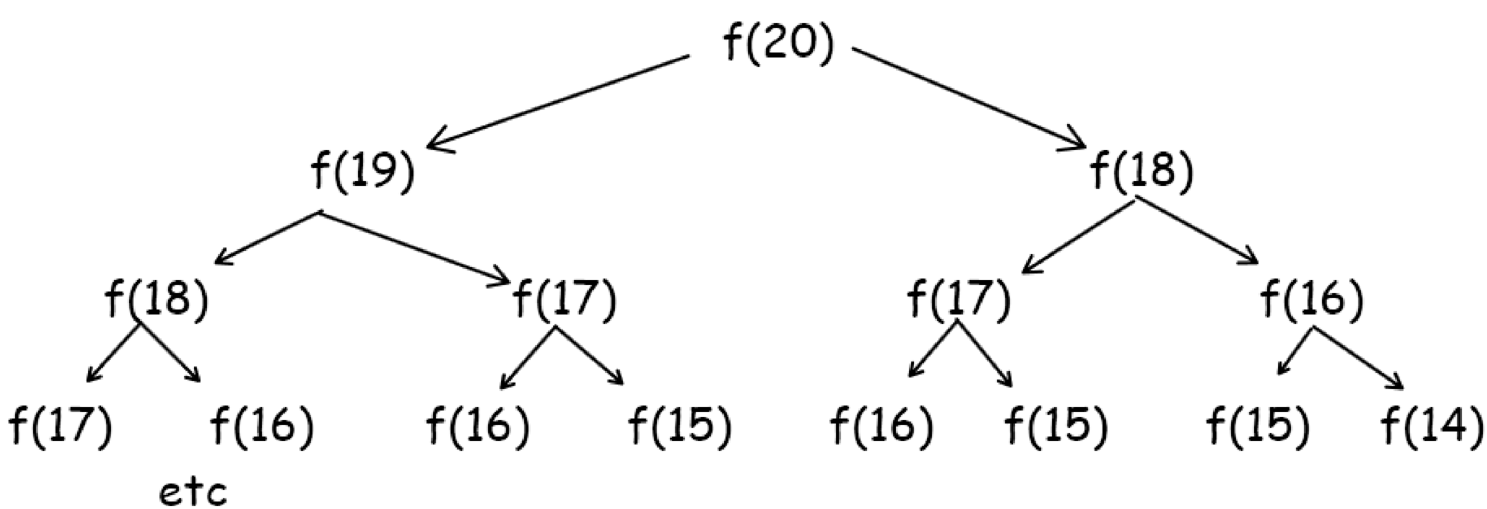
- You gotta believe
  - Trust in the recursive solution
  - Just consider one step at a time

What the Fib?

- The nth Fibonacci number F(n) is defined by:

  $F(n) = F(n-1) + F(n-2)$ for $n > 1$

  $F(1) = F(2) = 1$
- Why does recursive Fibonacci take so long?!?
  - Hint: How deep is the right-most branch of the tree below? Hence how big the tree? Hence how long does the computation take?
- How can we fix it?
  - Use a memory table! Same idea as what some of you noticed about Ackermann, but more powerful with Fibonacci.



Q1-2

Memory tables

- To speed up the recursive calculation of the nth Fibonacci number, just:
  1. "Memorize" every solution we find to subproblems, and
  2. Before you recursively compute a solution to a subproblem, look it up in the "memory table"
     - So to compute the nth Fibonacci number, construct an array that has n+1 elements, all initialized to 0. Then call Fib(n).
     - The base case for Fib(k) remains the same as in the naive solution.
     - At the beginning of the recursive step computing Fib(k), see if the k^{th} entry in the array is 0.
       - If it is NOT 0, return it.
       - If it IS 0, compute Fib(k) recursively. Then store the computed value in the k^{th} spot of the array. Then return the computed value.

This is a classic ***time-space tradeoff***

- A deep discovery of computer science
- Studied by "Complexity Theorists"
- Used everyday by software engineers

Tune the solution by varying the amount of storage space used and the amount of computation performed

Q3

Two Mirrors

If you actually do this, what really happens is Douglas Hofstadter appears and talks to you for eight hours about strange loops.

Mutual Recursion

- Two or more methods that call each other repeatedly
  - For example, Hofstadter Female and Male Sequences

$$F(n) = \begin{cases} 1 & \text{if } n = 0 \\ n - M(F(n-1)) & \text{if } n > 0 \end{cases}$$

$$M(n) = \begin{cases} 0 & \text{if } n = 0 \\ n - F(M(n-1)) & \text{if } n > 0 \end{cases}$$

  - Questions:
    - How often are the sequences different in the first 50 positions? first 500? first 5,000? first 5,000,000?

Q4

What is sorting?

- Let’s see…

Why study sorting?

- Shlemiel the Painter

What makes a program "good"?

- Correct – meets specifications
- Easy to understand, modify, write
- Uses reasonable set of resources
  - Time (runs fast)
  - Space (main memory)
  - Hard-drive space
  - Peripherals
  - …
- Here we focus on "runs fast" – **how much CPU time does the program / algorithm / problem take?**
  - Others are important too!

Course Goals for Sorting: You should…

- Be able to describe basic sorting algorithms:
  - Selection sort
  - Insertion sort
  - Merge sort
  - Quicksort
- Know the run-time efficiency of each
- Know the best and worst case inputs for each

Selection Sort

- Basic idea:
  - Think of the list as having a sorted part (at the beginning) and an unsorted part (the rest)
  - Find the smallest number in the unsorted part
  - Move it to the end of the sorted part (making the sorted part bigger and the unsorted part smaller)

Repeat until unsorted part is empty

Profiling Selection Sort

- Profiling: collecting data on the run-time behavior of an algorithm

- How long does selection sort take on:
  - 10,000 elements?
  - 20,000 elements?
  - …
  - 100,000 elements?

Q5-6

Big-Oh motivation: why profiling is not enough

- Results from profiling depend on:
  - Power of machine you use
    - CPU, RAM, etc
  - Operating system of machine you use
  - State of machine you use
    - What else is running? How much RAM is available? …
  - What inputs do you choose to run?
    - Size of input
    - Specific input

Big-Oh motivation: what it provides

- Big-Oh is a mathematical definition that allows us to:
  - Determine how fast a program is (in big-Oh terms)
  - Share results with others in terms that are universally understood
- Features of big-Oh
  - Allows paper-and-pencil analysis
  - Is much easier / faster than profiling
  - Is a function of the *size of the input*
  - Focuses our attention on *big* inputs
  - Is machine independent

Analyzing Selection Sort

- Analyzing: calculating the performance of an algorithm by studying how it works, typically mathematically
- Typically we want the relative performance as a function of input size

- Example: For an array of length *n*, how many times does **`selectionSort()`** call **`compareTo()`**?

Handy Fact

$$1 + 2 + \ldots + (n-1) + n = \frac{n(n+1)}{2}$$

Q7–12

Asymptotic analysis

- We care most what happens when **n** (the size of a problem) gets large
  - Is the function basically linear, quadratic, exponential, etc. ?
  - Consider: Why do we care most about large inputs?

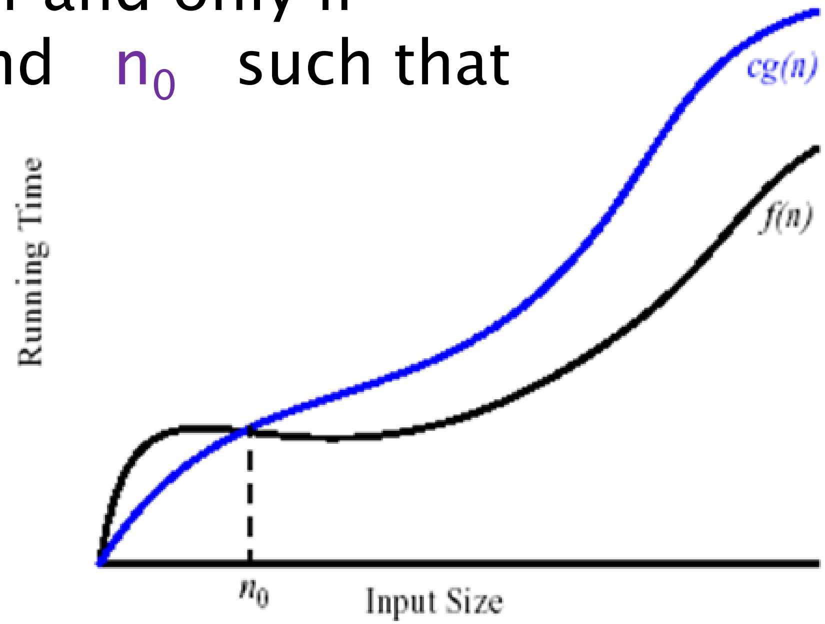
- For example, when **n** is large (or even moderate):
  - The difference between n^2 and $n^2 - 3$ is negligible.
  - n^3 is pretty large but 2^n is REALLY large.

- We say, "selection sort takes on the order of n^2 steps"

- Big-Oh gives a formal definition for
  "on the order of"

Definition of big-Oh

- Formal:
  - We say that <u>$f(n)$ is $O(\ g(n)\ )$</u> if and only if
  - there exist constants c and n_0 such that
  - for every $n \geq n_0$ we have
  - $f(n) \leq c \times g(n)$



- Informal:
  - $f(n)$ is *roughly proportional* to $g(n)$, for large n

- Example: $7n^3 + 24n^2 + 3000n + 45$ is $O(n^3)$
  - Because it is $\leq 3{,}077 \times n^3$ for all $n \geq 1$