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2008 Eur. J. Phys. 29 745

(http://iopscience.iop.org/0143-0807/29/4/009)

IOP PUBLISHING EUROPEAN JOURNAL OF PHYSICS

Eur. J. Phys. **29** (2008) 745–751 doi:10.1088/0143-0807/29/4/009

Magnetic braking revisited: activities for the undergraduate laboratory

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Received 17 April 2008
Published 9 June 2008
Online at stacks.iop.org/EJP/29/745

Abstract
This paper revisits the demonstration of Lenz by dropping magnets down a non-magnetic tube. Recent publications are reviewed and ideas for undergraduate laboratory investigations are suggested. Finally, an example of matching theory to observation is presented.

(Some figures in this article are in colour only in the electronic version)

Introduction

Demonstrating Lenz's law by dropping a neodymium magnet down a length of copper tube is not a new activity in any course which involves the use of Lenz's law. Indeed novel ways of using neodymium magnets have been recently published (Saravia 2006, Featonby 2006, Iniguez *et al* 2004, Roy *et al* 2007, Pelesko *et al* 2005). However, we offer here a twist to previous work by the use of *stacks* of magnets and by requiring students to predict trends from limited evidence. When observation does not match prediction, the opportunity for developing a mathematical model presents itself.

The investigation

Taking a length of copper tube and a stack of ten neodymium magnets, the following can be set up, as in figure 1(a). Students should record average fall and use their results to generate an average velocity, repeating the process with nine, eight, seven, six and five magnets in the stack.

Now ask the students to predict the fall-time and average velocity with four, three, two and one magnet, giving their reasons and supporting their arguments by generating suitable graphs.

An example graph using magnets of thickness 0.05 m and a tube length of 0.91 m is shown in figure 2.

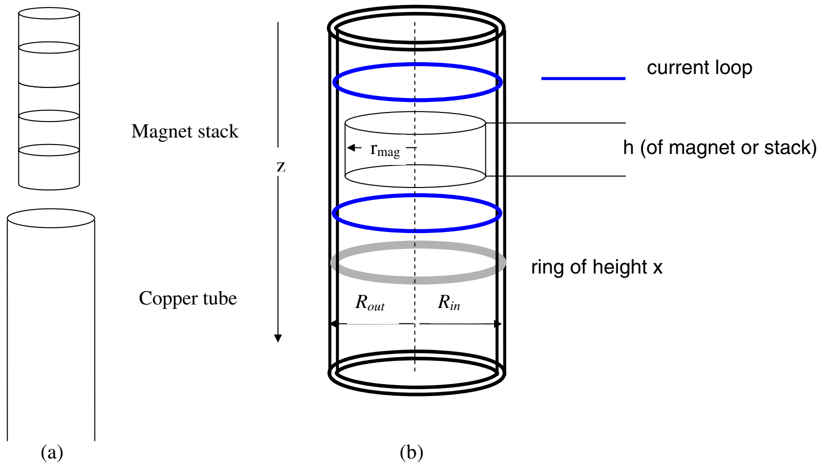

Figure 1. (a) Dropping a stack of neodymium magnets through a copper tube. (b) Reference diagram for equations used in the paper.

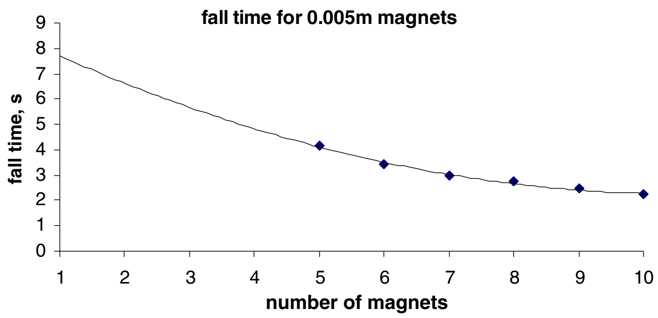

Figure 2. Fall time for 5 mm magnets in a 0.91 m tube and trend predicted for four, three, two and one magnets.

Table 1. Fall time for magnets of thickness 5 mm in a tube of length 0.91 m.

Number of magnets	Time to fall, s	Mean velocity, ms^{-1}
10	2.23	0.408
9	2.45	0.371
8	2.73	0.333
7	2.96	0.307
6	3.45	0.264
5	4.16	0.219
4	4.78	0.190
3	5.48	0.166
2	5.91	0.154
1	5.49	0.166

Students could now measure the fall times, calculate the average velocity and compare observed and expected values.

What is surprising, at least to the authors, is that a peak fall time or minimum velocity is recorded (see table 1).

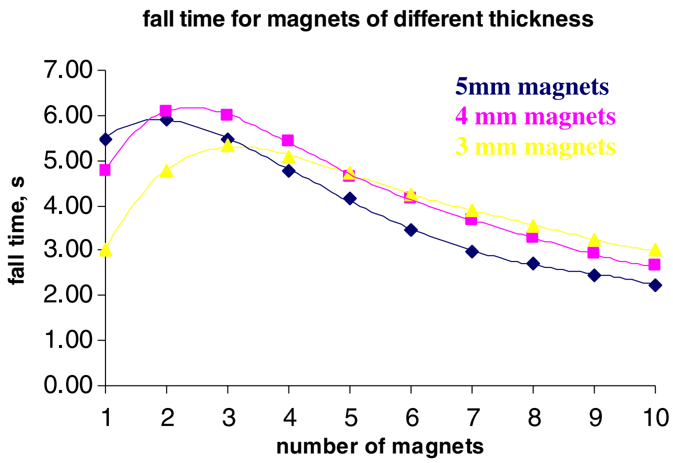

Figure 3. Fall time for magnets of different thickness.

Table 2. Fall time for magnets of different thickness.

	Fall time (s)		
Magnets	5 mm	4 mm	3 mm
10	2.23	2.66	3.00
9	2.45	2.92	3.25
8	2.73	3.27	3.56
7	2.96	3.68	3.90
6	3.45	4.15	4.24
5	4.16	4.65	4.72
4	4.78	5.44	5.09
3	5.48	6.00	5.33
2	5.91	6.09	4.78
1	5.49	4.76	3.03

This may be considered by the students, not unreasonably, to be linked to the number of magnets in the stack. The investigation can be repeated with magnets of the same diameter but different height. This shows that, whilst this is obviously a factor, it is not the same number of magnets of each height which gives the maximum fall time but rather the pole separation of the stack (see table 2). This effect can be seen better if the results are graphed (see figures 3 and 4).

Towards an explanation

When considering the falling magnet in a conducting tube the equation of motion can be considered as a damped motion allowing us to write

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - kv, \tag{1}$$

where m and v are the mass and velocity of the magnet(s), g is the acceleration due to gravity and k is a damping coefficient.

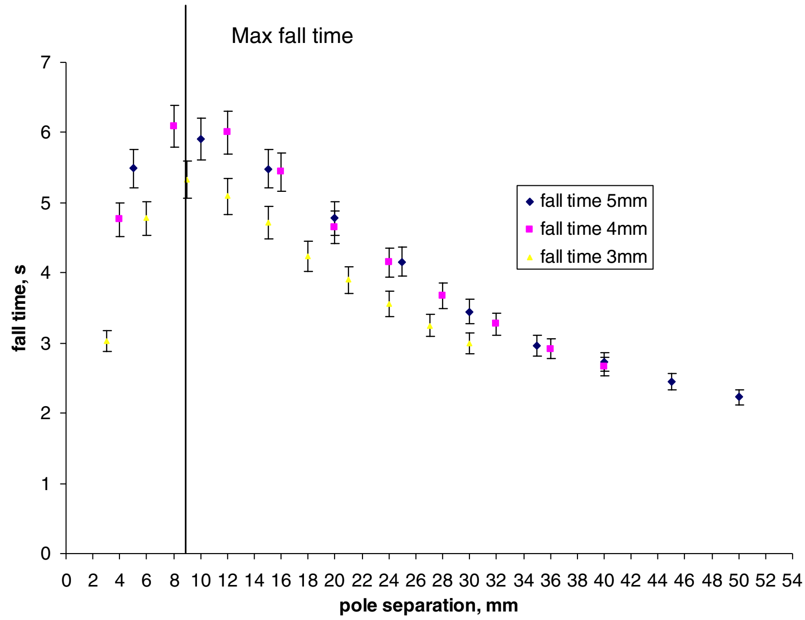

Figure 4. Fall time versus height of magnet stack or pole separation.

Integration of equation (1) allows us to write

$$v(t) = \frac{mg}{k}\left[1 - e^{\frac{-kt}{m}}\right], \tag{2}$$

which, following Iniguez *et al* (2004), allows us to write the time taken to reach terminal or asymptotic speed, τ, as

$$\tau = \frac{m}{k} \qquad \text{where} \quad k \approx \frac{mg}{v}. \tag{3}$$

Whilst Iniguez *et al* (2004) give a value of $\tau \approx 0.081$ s, values found during this investigation generate values between 0.015 s and 0.042 s. However, finding an expression for k from which predictions can be made is more difficult. Linking the geometrical and electromagnetic properties to the velocity of the falling magnet, Iniguez *et al* (2004) offer an expression for k, the damping coefficient, which can be written in the form (see figure 1(b))

$$k = \frac{\sqrt{2}}{\pi^2}\phi_0^2\sigma\sqrt{\frac{\ln^3(r_{out}/r_{in})}{r_{out}^2 - r_{in}^2}}, \tag{4}$$

where r_{in} and r_{out} are the inner and outer radii of the tube, as in figure 1(b), giving $r_{out} - r_{in} = t$, and σ is the conductivity of the tube and Φ_o the maximum flux.

Using a single magnet of mass 3 g, height 5 mm, radius 4.5 mm and Φ_o measured to be 27 μWb with a copper tube,

$r_{in} = 6$ mm
$r_{out} = 7.5$ mm
$\sigma = 59.10^6\ \Omega^{-1}\ \text{m}^{-1}$.

The value of k can be found and hence v can be calculated:

$k = 0.15$
$v = 0.19\ \text{ms}^{-1}$.

Table 3. Measured and predicted velocities using equation (4).

Measured v (ms^{-1})	Predicted v (ms^{-1})
0.17	0.19
0.15	0.27
0.17	0.38
0.19	0.44
0.22	0.59
0.26	0.71
0.31	0.82
0.33	0.94
0.37	1.10
0.41	1.18

Using this value for k, however, does not allow us to predict the velocity for more than one magnet in the stack, as shown in table 3.

Levin *et al* (2006), however, make the approximation that the flux can be treated as being due to two discs of radius r_{mag} separated by a distance h. By replacing the discs with point monopoles of the same net charge q_m, the flux through a ring a distance z from the nearest monopole can be given by

$$\Phi_z = \frac{\mu_0 q_m}{2}\left[\frac{z+h}{\sqrt{(z+h)^2 + r_{\mathrm{in}}^2}} - \frac{z}{\sqrt{z^2 + r_{\mathrm{in}}^2}}\right], \tag{5}$$

where μ_{o} is the permeability of free space.

Given that the magnet or stack of magnets is falling, the flux will change and give rise to an emf, ε_z, which from Faraday's law can be written as

$$\varepsilon_z = -\frac{\mathrm{d}\Phi_z}{\mathrm{d}t}. \tag{6}$$

Hence if the resistance, R, of the ring in which the current flows (assuming the length of the tube to be divided into rings each of height x and that the resistivity of the copper is ρ) can be given by

$$R = \frac{2\pi r_{\mathrm{in}}\rho}{tx}, \tag{7}$$

the current, I, flowing in a ring can be found using $\varepsilon = IR$. Combining equations (5) and (6), this is

$$I_z = \frac{\mu_0 q_m r_{\mathrm{in}}^2 v^2}{2R}\left[\frac{1}{\left(z^2 + r_{\mathrm{in}}^2\right)^{3/2}} - \frac{1}{\left[(z+h)^2 + r_{\mathrm{in}}^2\right]^{3/2}}\right], \tag{8}$$

where v is the terminal velocity of the falling magnet or magnets.

If we assume that the magnets are falling under gravity then at terminal velocity we may assume that the rate of change of gravitational potential energy is equal to the rate of dissipation of electrical energy, or

$$\frac{\mathrm{d}PE}{\mathrm{d}t} = \sum_z I_z^2 R, \tag{9}$$

which, following Levin *et al* (2006), can be written as

$$mgv = \sum_z I_z^2 R. \tag{10}$$

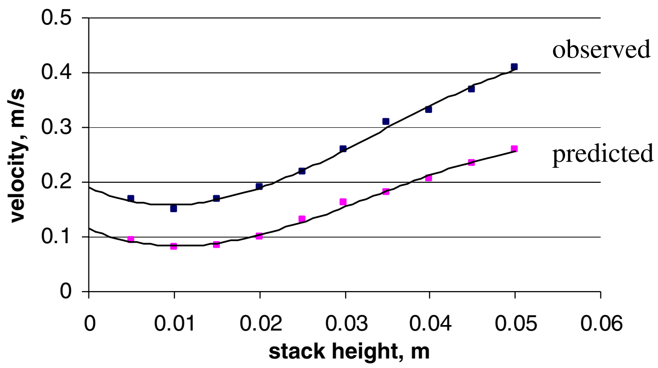

Figure 5. Observed and predicted velocity trends.

Table 4. Measured and predicted velocities using equation (12).

Number of magnets	Total mass m (kg)	B (Tesla)	Predicted v (ms^{-1})	Measured v (ms^{-1})
1	0.003	0.43	0.09	0.17
2	0.006	0.52	0.08	0.15
3	0.009	0.54	0.09	0.17
4	0.012	0.58	0.10	0.19
5	0.015	0.56	0.13	0.22
6	0.018	0.56	0.16	0.26
7	0.021	0.56	0.18	0.31
8	0.024	0.56	0.21	0.33
9	0.027	0.56	0.23	0.37
10	0.030	0.56	0.26	0.41

Since, in general, I^2R is electrical power, the right-hand side of equation (10) can be evaluated to give the power, P, as

$$P = \frac{\mu_0^2 q_m^2 r_{\text{in}}^4 v^2}{4R} \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{x} \left[\frac{1}{(z^2 + r_{\text{in}})^{3/2}} - \frac{1}{\left[(z+h)^2 + r_{\text{in}}^2\right]^{3/2}} \right]^2. \tag{11}$$

Combining equations (7), (10) and (11) allows us to write

$$v = \frac{8\pi m g \rho r_{\text{in}}^2}{\mu_0^2 q_m^2 t f\left(\frac{h}{r_{\text{in}}}\right)}, \tag{12}$$

where

$$f(x) = \int_{-\infty}^{\infty} \mathrm{d}y \left[\frac{1}{(y^2+1)^{3/2}} - \frac{1}{[(y+x)^2+1]^{3/2}} \right]^2 \tag{13}$$

$$q_m = \frac{2\pi B r_{\text{mag}}^2 \sqrt{h^2 + r_{\text{mag}}^2}}{\mu_0 h} \tag{14}$$

and[3] m is the total mass of magnets in the stack. Applying equation (12) to a stack of 10 magnets, the results shown in table 4 are obtained.

Whilst the numerical agreement is less than ideal we do, at least, have the same trend (see figure 5).

[3] To measure B the authors used the GM08 Gaussmeter from Hirst magnetic Instruments Ltd, Cornwall, England.

Conclusion

Obviously this is not the final solution but the results presented here do support those of Iniguez *et al* (2004), Roy *et al* (2007) and Pelesko *et al* (2005).

For the student it is hoped that this can serve as an exercise, of varying demand, going from the simple prediction based on limited evidence, through the model for a single magnet to the final model presented here.

We would further hope that readers may try this with their students using a wider range of diameters, thicknesses and lengths for the magnets and tubes. The authors would be pleased to receive any such data to further test their mathematical model.

Acknowledgment

The authors would like to acknowledge the contribution of Yan Levin, Instituto de Fisica, Universidade Federal do Rio Grande do Sul, Brazil, for the solution to equation (13) over the range of h:r_{in} ratios used in this investigation.

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