Homework 3

Due: 27 September 2019

1) SVD practice with non-square matrices

Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} \mathbf{4} & \mathbf{3} \\ \mathbf{1} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} \mathbf{837/712} & \mathbf{125/267} \\ \mathbf{125/267} & \mathbf{587/712} \end{bmatrix}$$

a) Determine the rank of each matrix. Compute (by hand) the SVD for each matrix and check the MATLAB answer.

b) Explain the geometric properties (sketch ellipsoids, etc.) of the linear equation **y=Ax** where **x** and **y** have the appropriate dimensions, in terms of the singular values and the singular vectors (defined by the unitary matrices **U** and **V**).

c) repeat part b) for the linear equation **y=Bx**.

d) Use the SVD to compute the pseudo-inverse of **A** and **B.**

2) SVD for real matrix

Consider the linear equation: **y=Ax** (Z)

where the real matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{7} \\ \mathbf{1} & \mathbf{3} & \mathbf{7} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix}$$

a) Carry out the singular value decomposition (SVD) of **A** using the MATLAB software of the form

$$\mathbf{A}=\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{T} \qquad (Y)$$

and state the numerical values of the matrices in Eq. (Y). Provide a brief geometrical description, including a rough 3-D sketch, of the transformation defined by Eq (Z).

Also, find the SVD of $\mathbf{A}^{-1}$ without explicitly computing it.

b) Let $\mathbf{u}_i$ and $\mathbf{v}_i$, i = 1,2,3, denote the column vectors of the matrices **U** and **V**, respectively, in Eq. (Y). Suppose that

$$\mathbf{x}=\mathbf{v}_1+2\mathbf{v}_2+3\mathbf{v}_3$$

then find **y** using the singular values fo **A** and the vectors $\mathbf{u}_i$. Next suppose that

$$\mathbf{y}=\mathbf{u}_1+\mathbf{u}_2+3\mathbf{u}_3$$

then find **x** using the singular values of **A** and the vectors $\mathbf{v}_i$