Homework 2

Due: Friday, 20 September 2019

1) Find the closed-form solution to the forced initial value problem

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_1, \qquad x(0) = \begin{Bmatrix} -1 \\ 0 \\ 1 \end{Bmatrix}$$

Plot your closed-form solution and compare to the numerical solution obtained using the Matlab 'lsim' command.

HINT: use the technique from Burchett section 7.10.

2) Given the following state space model from class:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u; \quad \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{Bmatrix} = \begin{bmatrix} -A & 0 & 0 & -D & 0 & 0 \\ 0 & -A & D & 0 & 0 & 0 \\ 0 & C & E & -F & 0 & 0 \\ -C & 0 & F & E & 0 & 0 \\ 0 & 0 & 0 & 0 & H & J \\ 0 & 0 & 0 & 0 & T & L \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ G & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & M \\ 0 & N \end{bmatrix} \begin{Bmatrix} g \\ 1 \end{Bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}; \quad \mathrm{D} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

a) Is the system controllable? Explain why or why not. b) Is the system observable? Explain why or why not. c) Suppose the eigenvalues of $\begin{bmatrix} \boldsymbol{H} & \boldsymbol{J} \\ \boldsymbol{T} & \boldsymbol{L} \end{bmatrix}$ are $s_1 = \mathbf{-0.0049}$ and $s_2 = \mathbf{-0.02226}$. Is the system detectable?

3) Given the following equation of motion:

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{Bmatrix} = \begin{bmatrix} 0.0000000 & 0.00000 & 0.0000 & 1.000 & 0.000 & 0.000 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\dfrac{k_1 + k_2}{m_1} & \dfrac{k_2}{m_1} & 0 & -\dfrac{b_1}{m_1} & 0 & 0 \\ \dfrac{k_2}{m_2} & -\dfrac{k_2 + k_3}{m_2} & \dfrac{k_3}{m_2} & 0 & -\dfrac{b_2}{m_2} & 0 \\ 0 & \dfrac{k_3}{m_3} & -\dfrac{k_3}{m_2} & 0 & 0 & -\dfrac{b_3}{m_3} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} F,$$

Take $b_1 = b_2 = b_3 = 0$, $k_1 = k_2 = k_3 = 1$, and $m_1 = m_2 = m_3 = 1$.

a) Use Matlab to compute the eigenvalues and eigenvectors of **A.**
b) Set the initial condition equal to an eigenvector, and use the **initial** command to simulate the response. For each mode, plot all three position states on a common axis. Comment on the frequency of ensuing oscillation compared to the eigenvalue of the corresponding mode. Describe the relative motion of each mode.