# Homework 1

**Due: Friday, 13 September 2019**

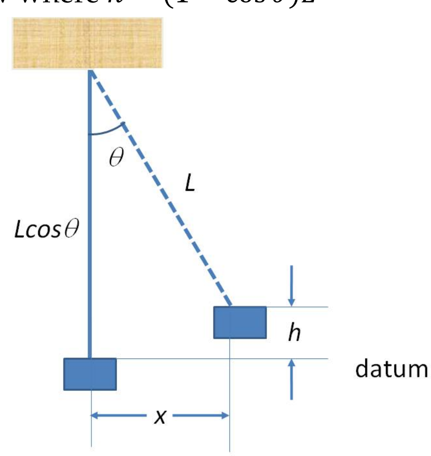
**Day 1**

1) Given the following non-linear equation for the pendulum system, use the Taylor series method to linearize the equation about the operating points $\theta = 0^\circ$ , $\dot{\theta} = 0$ (crane mode), and $\theta = 180^\circ$ , $\dot{\theta} = 0$ (inverted mode). Write each set of equations in state-space form, determine the eigenvalues, and comment on stability of the equilibrium point.

$$\ddot{\theta} + \dot{\theta} + \sin\theta = 0$$

2) Given the ballistic pendulum schematic below where $h = (1 - \cos\theta)L$

   Use the Taylor series expansion to show that $h$ can be approximated as: $h \approx \frac{x^2}{2L}$
   HINTS: Write $\sin\theta = \frac{x}{L}$, then use the Pythagorean theorem to get $\cos\theta = f(x)$. Subs the result into the original expression for $h$. Expand the Taylor series out to the second order term and evaluate at $x = 0$.



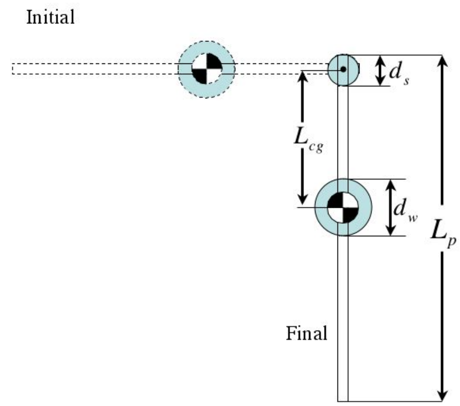
3) The pendulum with adjustable added mass is released from rest near the horizontal and swings to the vertical. Numerical values are given below.

Find the non-linear equation of motion, and linear perturbation model (Jacobian matrices) leaving $\theta$ as a symbol. Use Simulink to simulate the following three models and compare for accuracy:

a) Full non-linear simulation

b) Linearized model, using $\theta = 0$ as expansion point.

c) A time varying pseudo-linear model where the Jacobian matrix is augmented with a non-linear term at the current state, as:




$$\left\{ \begin{array}{c} \dot{\mathbf{x}} \\ \dot{1} \end{array} \right\} = \left[ \begin{array}{cc} \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\Big|_{\bar{\mathbf{x}},\bar{\mathbf{u}}} & \mathbf{f}(\bar{\mathbf{x}}) \\ \mathbf{0} & 0 \end{array} \right] \left\{ \begin{array}{c} \delta\mathbf{x} \\ 1 \end{array} \right\} + \left[ \begin{array}{c} \frac{\partial \mathbf{f}}{\partial \mathbf{u}}\Big|_{\bar{\mathbf{x}},\bar{\mathbf{u}}} \\ 0 \end{array} \right] \delta\mathbf{u}$$

| Parameter | Value with units |
|---|---|
| $m_p$ | 68.5 g |
| $m_w$ | 88 g |
| $l_p$ | 43.2 cm |
| $d_w$ | 5 cm |
| $d_s$ | 2.5 cm |

For part c) Show results updating the Jacobian matrix every 15 degrees and every 30 degrees. Note that you must form the perturbation state as $\delta\theta = \theta - \bar{\theta}$. The "Quantizer" block in simulink is helpful in determining the trim point.