ROSE-HULMAN INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

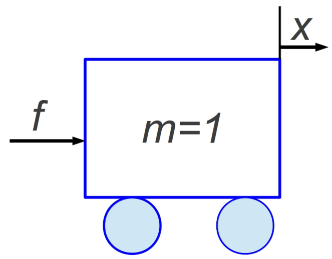
ME 506 Advanced Controls

Homework 8: **Deadline: Nov 13, 2019**

1. Write a Matlab code to demonstrate Pseudo-spectral solution of the point mass vacuum trajectory as shown in class. Use the Lagrange-Gauss-Lobatto collocation points. For a launch velocity of 100 mph, plot trajectories for launch angles from 15 to 75 degrees in increments of 15 degrees. Plot continuous lines using 34 collocation points and then overlay the solution using only 4 point with discrete markers.

2. Write a Matlab code to demonstrate the Gauss Pseudo-spectral solution of the following linear optimal control problem:

A mass is moved on a frictionless surface from an initial point x_0 to a desired point x_f.



The equation of motion is simply:

$$m\ddot{x} = f(t) \tag{1}$$

Convert this second order equation to a matrix set of first order equations in the form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}f \tag{2}$$

Where. I.E.

$$\begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} \\ \end{bmatrix} f \tag{3}$$

Use the Lagrange-Gauss-Lobatto collocation points to convert Eq. (3) to a set of linear algebraic equations. Find the trajectory and control which minimize the cost function:

$$J = \frac{1}{2}\mathbf{x}^T\mathbf{S}\mathbf{x} + \frac{1}{2}\int_0^{t_f} \dot{x}^2 + u^2 \, dt \tag{4}$$

Where

$$\mathbf{S} = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}$$

Note that the integration in Eq. (4) is to be done by a Gauss quadrature within the cost FUN used by fmincon.

Use 20 collocation points for the optimization. Plot the cart position and velocity, and the control effort for the optimal trajectory. Use m =1, x_0 = 1, x_f = 0, and t_f = 5.

3. Write a Matlab code to demonstrate an approximate solution to the brachistochrone problem using the Gauss Pseudo-spectral method.

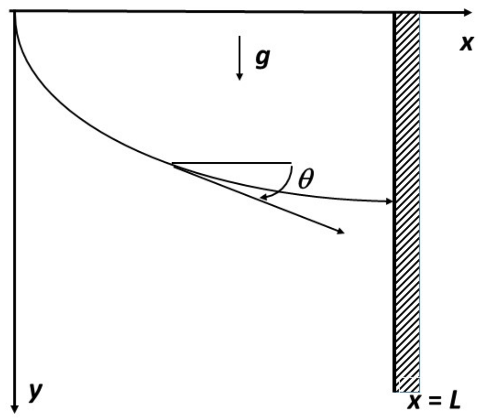


Figure 1: Brachistochrone Problem

The objective is to determine the optimal path that a bead would take sliding down a frictionless wire to reach a given horizontal position in the minimum possible time [1].

The Eqs. of motion are

$$\dot{x} = \sqrt{2gy}\ \cos\theta \qquad \dot{y} = \sqrt{2gy}\ \sin\theta \tag{4}$$

With initial conditions:

$$x(0) = 0 \qquad y(0) = 0 \tag{5}$$

And terminal condition:

$$x(t_f) = L \tag{6}$$

The true solution for the control angle is

$$\theta(t) = \frac{\pi}{2} - \omega \cdot t \tag{7}$$

Where

$$\omega = \sqrt{\frac{\pi \cdot g}{4 \cdot L}} \qquad t_f = \sqrt{\frac{\pi \cdot L}{g}} \tag{8}$$

The optimal trajectory is:

$$x(t) = \frac{2 \cdot L}{\pi}\left(\omega \cdot t - \frac{\sin(2 \cdot \omega \cdot t)}{2}\right) \tag{9}$$

$$y(t) = \frac{2 \cdot L}{\pi}\ (\sin(\omega \cdot t))^2 \tag{10}$$

To overcome the solution singularity at the initial condition, start your discretized solution at

$$x(0) = 0.01 \qquad y(0) = 0.01 \tag{11}$$

Use the Legendre-Gauss differentiation matrix and the LGL collocation points. Note that the cost function is the final time, and the final state must be constrained by a non-linear constraint such that the bead reaches the desired location at the final time. This can be coded using a Gauss quadrature:

$$\frac{t_f}{2} w^T \cdot \left(\sqrt{2 \cdot g \cdot Y_N} \cdot \cos\theta_N\right) - L = 0 \tag{12}$$

Use fmincon to optimize. Use 10 or more collocation points, and take $g = 1$, $L = 1$. Note that the differential constraints and Eq. (12) must be placed in a separate NONLCON function.

Plot your numerical solution using discrete markers in (x, y) space. Overlay the exact solution as a continuous line.