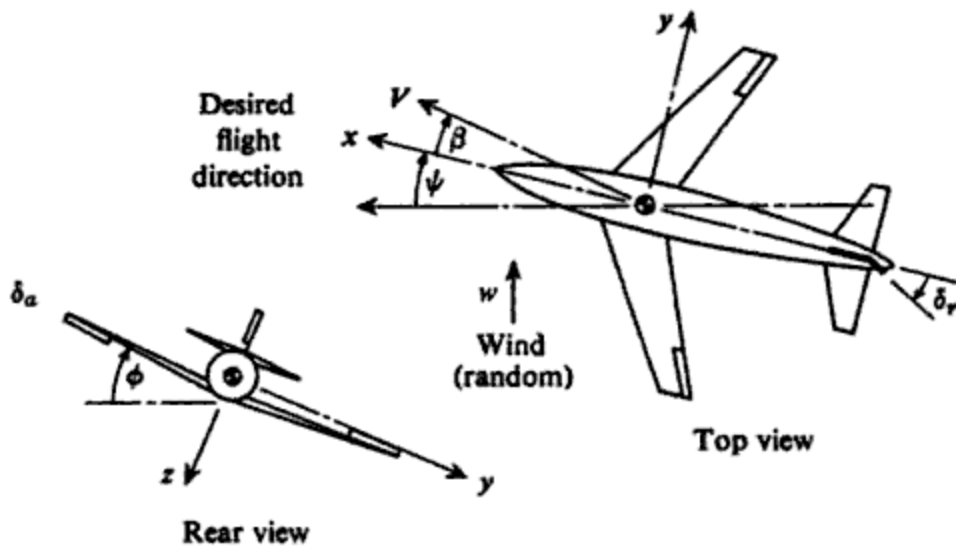


Homework 7 Due 4 November 2019 at 4:20 PM**1. LQR Control of the Lateral Dynamics of a Transport Aircraft.**

In this problem set, you are given a model of the lateral dynamics of a jet transport aircraft and you are asked to design several auto-pilots. A stochastic (disturbance) side wind gust acts on the aircraft; you will simulate the aircraft response open and closed loop. Finally you are asked to evaluate the loops in a deterministic setting.



The lateral motions of an airplane (roll, yaw, sideslip) are negligibly coupled to the longitudinal motions (pitch, angle of attack, speed). A typical jet transport aircraft weighing 100,000 pounds, flying at 30,000 feet altitude and a velocity of $V = 500$ miles per hour has the following lateral equations of motion. These equations result from linearizing the general (non-linear) equations for side force, yaw moment, and roll moment about the straight and level flight conditions ($\phi \ll 1$, $\beta \ll 1$).

The state variables are all measured in radians or radians / sec and they are:

$\beta(t)$: sideslip angle (the angle between the fuselage and the velocity vector)

$r(t)$: the yaw rate (time derivative of yaw angle ψ)

$p(t)$: the roll rate (time derivative of the roll- or bank - angle ϕ)

$\phi(t)$: the roll angle

$\psi(t)$: the yaw angle

The control variables are also measured in radians and they are:

$\delta_r(t)$: the rudder angle

$\delta_a(t)$: the aileron angle

The disturbance is generated by wind turbulence. For the model that we shall use, it is convenient to normalize the effect of the wind disturbance as an equivalent wind-angle $w(t)$ perturbation to the sideslip angle $\beta(t)$. The wind-angle $w(t)$ has the units of radians and is related to the actual wind velocity as follows:

$$w(t) = \frac{w_v(t)}{V}$$

Where $w_v(t)$ is the side wind gust (in miles per hour) and V is the aircraft speed (in miles per hour). The differential equations of motion using standard aero representation are as follows:

$$\dot{\beta}(t) + r(t) = -0.0297(\beta(t) - w(t)) + 0.0438\Phi(t)$$

$$\dot{r}(t) - 0.0423 \dot{p}(t) = 0.3790(\beta(t) - w(t)) - 0.0096r(t) - 0.0125p(t) - 0.3790\delta_r(t)$$

$$\dot{p}(t) - 0.1060 \dot{r}(t) = -1.170(\beta(t) - w(t)) + 0.1290r(t) - 0.7900p(t) + 1.580\delta_a(t)$$

$$\dot{\Psi}(t) = r(t)$$

$$\dot{\Phi}(t) = p(t)$$

The wind-angle disturbance dynamics are modeled by a first order Markov process as follows:

$$\dot{w}(t) = -0.1w(t) + 0.0224\xi(t)$$

where $\xi(t)$ is scalar zero-mean, unit intensity, continuous-time white noise. This stochastic model for the (serious) wind gusts corresponds to an RMS wind velocity of about 25 mph with a time constant of 10 secs.

Output quantities of interest are as follows. The heading angle $\chi(t)$ represents the desired direction of the velocity vector, measured from some arbitrary reference (see figure), and is defined by

$$\chi(t) = \psi(t) + \beta(t)$$

We are also concerned with the lateral acceleration $n_y(t)$ – which makes it rough walking in the aisle – which is measured in g s and defined by (g is the acceleration of gravity)

$$n_y(t) = \frac{V(\dot{\beta}(t) + r(t))}{g} - \sin\phi(t)$$

which for small angles, i.e. $\sin\phi \approx \phi$, reduces to

$$n_y(t) = -0.0118(\beta(t) + w(t))$$

Open loop Dynamics

Write the state equations in the form

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}\xi(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned}$$

where the state vector is $\mathbf{x} = [\beta \ r \ p \ \phi \ \psi \ w]^T$ the control vector $\mathbf{u} = [\delta_p \ \delta_a]^T$ and the output vector is $\mathbf{y} = [\chi \ n_y]^T$ and change all angle units to degrees and degrees / sec.

What are the eigenvalues, eigenvectors, and modes? (Note that two of the modes form a complex pair called the "Dutch roll" mode (resembling the skating of a fat Dutchman?) Briefly describe the motion associated with the Dutch roll mode and identify the other three modes as "yaw integrator", "spiral", "roll damping", and "wind".

Plot the singular values vs frequency of the open loop transfer function matrix

$$y(s) = G(s)u(s); G(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

and determine its transmission zeros. Also, plot the Bode magnitude of the open-loop transfer function from $w(s)$ to $n_y(s)$.

Identify all obvious poles and zeros in these frequency plots.

Let $\mathbf{u}(t) = \mathbf{0}$. Also eliminate the yaw angle ψ from the state equations, so that the open loop dynamics are strictly stable.

Carry out a stochastic open loop time-domain simulation for about 200 seconds and plot the time trajectories of all state variables and of the lateral acceleration n_y .

To do this you have to accurately approximate the continuous-time white noise $\xi(t)$ process on the digital computer, and this can be somewhat tricky. The MATLAB function **randn** generates discrete white gaussian noise, with zero mean, and unit variance. Since white noise has infinite variance, you should multiply the discrete white noise by $(1/\sqrt{\Delta t})$, where Δt is the integration step-size. Thus, the white noise driving signal for the system can be created in MATLAB with: $xi=randn(size(t))/sqrt(dt)$.

Deterministic Autopilot Regulator Designs

Set $w(t) = 0$.

An autopilot for lateral control would regulate the heading and roll angles without subjecting the passengers to excessive lateral accelerations. We shall use the following structure of the quadratic cost functional.

$$J = \int [\chi^2(t) + a\phi^2(t) + bn_y^2(t) + \rho(\delta_r^2(t) + \delta_a^2(t))]dt \quad (10)$$

Design A: Here we only penalize heading errors and bank angle. So let $a = 1/9$, $b=0$, and $\rho = 1/2$ in eq 10. Determine the optimal LQR design and compare the time-domain transients between open-loop and closed-loop for all state, output, and control variables for the following initial conditions

$$\begin{aligned} \psi(0) &= 90^\circ, \text{ all other initial states} \\ &\text{zero} \\ \phi(0) &= 60^\circ, \text{ all other initial states zero} \end{aligned} \quad (11)$$

Also determine and plot the closed-loop poles. Pay special attention to the lateral acceleration transient.

Design B: In this design we penalize heavily lateral acceleration, thus forcing the aircraft to execute a coordinated turn. In reference to eq. (5) the first term represents a centrifugal acceleration while the second term captures gravity. In a coordinated turn, the acceleration forces the passengers into their seats, resulting in a more comfortable ride. So we modify Design A by having $a = 1/9$, $b = 10^6$, $\rho = 1/2$ in eq. (10).

Determine the optimal LQR design and compare the time-domain transients between open-loop and closed-loop for all state, output and control variables for the following initial conditions.

Also determine and plot the closed-loop poles. Pay special attention to the lateral acceleration transient, and compare with Design A.

Stochastic Evaluation of Design B.

In the above designs we have neglected the wind disturbance. Consider Design B, and evaluate the properties of the closed-loop regulator in the presence of disturbance, although the wind state $w(t)$ is not measured and it does not influence the optimal control.

Simulate for 200 seconds both the open loop and the closed-loop Design B to the same white noise and present comparisons for heading, roll, lateral acceleration, rudder and aileron. Use zero initial conditions for all states.

Also plot on a Bode plot the magnitude of the closed-loop transfer function

- (a). from white noise $\xi(t)$ to heading angle $\chi(t)$
- (b). from white noise $\xi(t)$ to roll angle $\phi(t)$
- (c). from white noise $\xi(t)$ to lateral acceleration $n_y(t)$