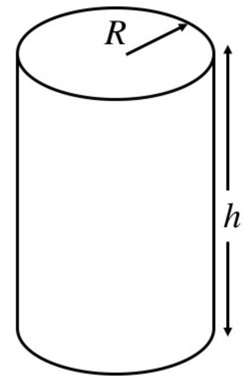
ROSE-HULMAN INSTITUTE OF TECHNOLOGY

Department of Mechanical Engineering

Homework 5, Due October 17, 2019

1. Maximize the volume of a cylindrical can of fixed cost c_0 cents, if the top and bottom cost c_1 per square inch and the side of the can costs c_2 cents per square inch.



2. LQR control of an underdamped SISO plant.
The purpose of this problem is to provide you analytical insight into some of the properties of LQR controllers in SISO applications.
The transfer function description of the plant is

$$G(s) = \frac{6543}{s^2 + 75.74s + 3137}$$

a. Find the state space description of the plant in control canonical form.
b. Set the control penalty **R** = 1, and state penalty **Q** = $\mathbf{C}^T\mathbf{C}$
c. Solve for the LQR gain and closed-loop eigenvalues
d. Set observer poles to $\lambda_{OBSV}(1)$=3*Re($\lambda_{LQR}(1)$), $\lambda_{OBSV}(2)$=5*Re($\lambda_{LQR}(1)$)
e. Find the observer gain using the eigenstructure assignment algorithm
    a. Explicitly show use of the 'svd' function to find the nullspace vectors
f. Convert compensator to TF using:

$$G_c(s) = \mathbf{K}\left(s\mathbf{I} - (\mathbf{A} - \mathbf{BK} - \mathbf{HC})\right)^{-1}\mathbf{H}$$

Repeat a – f varying **R** from 10^{-3}<**R**<2. Use 100 points evenly space in log (logspace). Plot the closed-loop poles on a single axis for all values of **R**.

Find a value of **R** that results in a closed-loop rise time of 10ms or less, and an overshoot of 5% or less. (Based upon location of the closed-loop poles). Determine the compensator and pre-filter transfer functions for this value of **R**. The prefilter will be a transfer function whose poles match the compensator zero(s), and whose gain sets the overall loop gain to unity. The loop architecture is shown below. Collect a simulated step response for the closed-loop system and verify that transient response specifications are met, and the final value is 1.