No books or notes are permitted. You may use your laptop and any resources on its C drive and a formula sheet attached as the last page of this test. You may detach the formula sheet and it need not be handed in. **No network connections are allowed.** Anyone with a physical connection to the network will receive an "F" on the test.

**Show all your work.** No credit given for answers appearing without supporting calculations. Partial credit may be given depending upon clarity and completeness of answers.

1. Light from a helium-neon laser ($\lambda = 633$ nm) travels downward along the y-axis with an $E$-field polarized in the $yz$-plane as shown in the figure.

(a) Write the letter corresponding to the expression below that best describes the wave.

- A. $E_y = E_0 \sin(kz - \omega t)$
- B. $E_y = E_0 \sin(ky - \omega t)$
- C. $E_y = E_0 \sin(kz - \omega t)$
- D. $E_y = E_0 \sin(kz + \omega t)$
- E. $E_y = E_0 \sin(ky + \omega t)$
- F. $E_y = E_0 \sin(kz + \omega t)$
- G. $E_y = E_0 \sin(ky - \omega t)$
- H. $E_y = E_0 \sin(ky + \omega t)$
- I. $E_y = E_0 \sin(kz - \omega t)$

(b) What is the numerical value for the propagation constant $k$ for this wave?

$$k = \frac{2\pi}{\lambda} = 9.93 \times 10^6 \text{ m}^{-1}$$

(c) What is the numerical value for the angular frequency $\omega$ for this wave?

$$\omega = \frac{c}{\lambda} = \frac{c}{7.99 \times 10^8} \text{ rad/s}$$

(d) If $E_0$ has magnitude 50.0 V/m, what is the amplitude of the magnetic field oscillation associated with this wave?

$$B_0 = \frac{E_0}{c} = 1.47 \times 10^{-7} \text{ T}$$

(e) In the figure above there is an arrow indicating the $z$-position where there is a peak in the electric field at a particular moment of time. Write an expression of the magnetic field vector (using $\hat{i}, \hat{j}, \hat{k}$ notation) at the same time and place. **[Partial credit for drawing the $B$-field lobe on the diagram.]**
2. A certain light bulb filament emits 20.0 W of electromagnetic radiation isotropically.

(a) What is the irradiance falling on a sheet of paper located a distance of 2.50 m from the filament?

\[ I = \frac{P}{A} = \frac{20 \text{ W}}{\frac{4\pi}{3} (2.50 \text{ m})^2} = \frac{0.255 \text{ W}}{\mu \text{m}^2} \]

(b) What is the amplitude of the electric field oscillations at this distance?

\[ E_0 = \sqrt{\frac{2P_0}{mc}} = 12.9 \ \text{V/m} \]

(c) How much electromagnetic energy is contained in a cube 1.00 cm on a side, if the cube is located a distance of 2.50 m from the filament. [If you could not do part (b), assume \( E_0 = 3.00 \ \text{V/m} \) at this distance and continue.]

\[ \langle \nu \rangle = \frac{1}{c} \frac{0.255 \text{ W/m}^2}{9\times10^8 \text{ W/m}^2} = 8.50 \times 10^{-10} \ \text{Hz} \]

\[ E = \langle \nu \rangle \times V = \left( 8.50 \times 10^{-10} \right) \left( 10^{-6} \text{ m} \right) \]

\[ = \frac{1}{8.50 \times 10^{-16} \ \text{J}} \]
3. As a physics project a student attempts to lift a small bit of paper using the power from a $40.0 \text{ mW}$ helium-neon laser. The laser beam is directed upward and focused down to a circular spot that is $0.200 \text{ mm}$ in diameter.

(a) What is the irradiance $I$ in the region where the beam is focused?

$$I = \frac{40 \text{ mW}}{\pi \left( \frac{0.200 \text{ mm}}{2} \right)^2} = \frac{40 \times 10^{-3} \text{ W}}{(0.1 \times 10^{-3} \text{ m})^2} = 1.27 \times 10^6 \text{ W/m}^2$$

(b) What is the radiation pressure on any object inserted into the beam?

$$P = \frac{I}{c} = \frac{4.7 \times 10^6 \text{ W/m}^2}{(2.998 \times 10^8 \text{ m/s})} \times \frac{N}{\text{m}^2}$$

(c) A small square of blackened paper measuring $0.100 \text{ mm}$ on a side is placed horizontally into the beam. What is the total upward force exerted on the paper by the beam?

$$F = Pa = P \left( \frac{4}{16} \right) = 4.7 \times 10^{-5} \text{ N}$$

(d) If the paper were replaced by a sheet of highly reflecting white paper, would the upward force be less, the same, or greater than on the black paper?

$$\text{Answer}$$
4. (a) Shown below are nine cases where a current, \( E \)-field, or \( B \)-field is directed normal to the plane of this paper. Below each diagram is a phrase indicating whether that current or field is increasing, decreasing, or remaining constant in time. Also on each diagram, is a dashed circle showing the possible locus of some type of induced field. Your task is, first, to determine whether there is an induced field, and, if there is, whether it is an \( E \)-field or a \( B \)-field. Second, if there is an induced field around the dotted circle, indicate the direction of that induced field. To illustrate how to mark your answers, the first of the nine cases is already worked.

(b) In the region between two circular plates of radius \( a = 2.00 \text{ cm} \), an electric field exists parallel to the \( x \)-axis as shown in the figure. [Assume the field exists only between the plates.] The magnitude of this field is given by

\[
\vec{E}(t) = [10.0 \text{ V/m} + (200 \text{ V/m/s}) t] \hat{i} 
\]

(i) Calculate the magnitude of the displacement current between the plates.

\[
\vec{D} = \varepsilon_0 \vec{E} = 2.22 \times 10^{-12} \text{ A} 
\]

(ii) Calculate the magnitude of the magnetic field in the \( yz \)-plane at a distance \( R = 4.00 \text{ cm} \) from the \( x \)-axis.

\[
B = \frac{\mu_0 I}{2\pi R} = \frac{(2\pi \times 10^{-7}) (2.78 \times 10^{-12})}{2\pi \times 4 \times 10^{-2}} = 1.11 \times 10^{-17} \text{ T} 
\]

\( \mu_0 \rightarrow 2\pi \times 10^{-7} \)