Problems

1) A beam is simply supported on each end with a spring connected to the center of the beam (other end of spring is fixed). Recognizing that the displacement “X” is measured from the static equilibrium point, replace the beam and spring with an equivalent spring and mass system (as illustrated below). The beam is 40 in. long, has an elastic modulus of $8 \times 10^6 \text{ lb/in}^2$, a weight of 4 lbf, the mass moment of inertia is $7.2 \times 10^{-3} \text{ in}^4$, and the spring stiffness coefficient is 450 lb/in.

\[ m_{eq} = 2 \text{ lbm}, \quad (62 \text{ mslugs}), \quad k_{eq} = 493 \text{ lb/in} \]

2) How would your results for problem 1 change, if at all, if the spring connected to the center of the beam has a mass connected to the other end instead of being fixed?

[spring constant for beam would be in series with $k$ not parallel, beam mass would be between $k$ and $k_{beam}$]

3) The diagram shown below should be used to answer the following questions. You may assume the mass moments of inertia ($J$), gear radii ($r$), applied torque ($T_A$), load torque ($T_L$) are given and that the gears are rigid and without backlash. The shaft between gears 1 and 2 is short and may be assumed ridged ($K_1 \rightarrow \infty$) while the shaft connecting gears 3 and 4 is long and therefore may not be assumed to be rigid ($K_2 < \infty$ and known).

\[ \Theta_1 \quad T_A \]
\[ \Theta_2 \]
\[ J_1 \quad K_1 \]
\[ J_2 \]
\[ \Theta_3 \quad T_L \]
\[ \Theta_4 \]
\[ J_3 \quad K_2 \]
\[ J_4 \]
\[ \Theta_5 \]

a) How many unknowns, constraints, and independent equations are there?

\[ 5, 3, 2 \text{ diff eqn} \]

b) Construct the FBD for the system. [diagrams]
c) Write the matrix form of the set of independent equations necessary to solve for unknowns.

\[
\begin{align*}
(J_1 + J_2) \ddot{\theta}_2 &= T_d - r_2 F_1 \\
J_3 \ddot{\theta}_3 &= r_3 F_1 - K_2 (\theta_3 - \theta_4) \\
\begin{bmatrix}
J_4 \ddot{\theta}_4 &= K_2 (\theta_3 - \theta_4) - r_4 F_2 \\
J_5 \ddot{\theta}_5 &= r_5 F_2 - T_L \\
\end{bmatrix} \\
\theta_i &= \theta_2, \quad r_2 \theta_2 = r_3 \theta_3, \quad r_4 \theta_4 = r_5 \theta_5
\end{align*}
\]

d) How will your solution change, if at all, if both connecting shafts are short and rigid?

\[
[\theta_3 = \theta_4, \quad K_2 \to \infty]
\]

4) Draw the FBD and generate the differential equations necessary to solve for the translation of \(x_1\), \(x_2\), and \(x_3\). You may assume that both \(f_1(t)\) and \(f_2(t)\) are given.

\[
\begin{align*}
m_1 \ddot{x}_1 &= k_1 (x_2 - x_1) - k_1 x_1 \\
\begin{bmatrix}
m_2 \ddot{x}_2 &= f_1 + k_3 (x_3 - x_2) + c (\dot{x}_3 - \dot{x}_2) - k_2 (x_2 - x_1) \\
m_3 \ddot{x}_3 &= f_2 - k_3 (x_3 - x_2) - c (\dot{x}_3 - \dot{x}_2) \\
\end{bmatrix}
\end{align*}
\]

5) Construct a simulink model for problem 4. [MATLAB simulink model]