Final Project: Design of a Rocket Launch Configuration

A small rocket has an initial total mass, \(m_0\), of 300 kg, which includes 180 kg of propellant. The rocket is to be launched from the ground level at an initial angle \(\gamma_0\) to the horizontal. After launch, the axis of the rocket is aligned with the flight path so that both the thrust \(T\) and the drag \(D\) are tangent to the flight path. The objective of the design is to configure the launch parameters so that the rocket has maximum (horizontal) range.

The equations of motion for the rocket are given in terms of the Cartesian \((x, y)\) coordinate system, with \(x\) denoting the horizontal (positive in the flight direction) and \(y\) denoting the vertical (positive upward). The following free-body diagram is included for an illustrative purpose.

With reference to the above free-body diagram, let \(u\) and \(v\) be the velocity components in the \(x\) and \(y\) directions, respectively, and let \(t\) denote time. Then we use the definition of velocity, Newton’s Second Law of motion, and the free-body diagram to write the system of governing equations as

\[
\frac{dx}{dt} = u \\
\frac{dy}{dt} = v \\
\frac{du}{dt} = \frac{(T - D)\cos \gamma}{m} \\
\frac{dv}{dt} = \frac{(T - D)\sin \gamma}{m} - g
\]

The flight path angle \(\gamma\) is related to the \(u, v\) velocity components by

\[
\gamma = \tan^{-1}\left(\frac{v}{u}\right)
\]
We shall assume that the propellant consumption rate $q$ varies linearly with time from the instant of launch ($t = 0$) to the burnout time $t_b$ (at which the propellant is completely consumed.) The linear model for the consumption rate is

$$q = q_0 + (q_b - q_0) \frac{t}{t_b} \quad \text{for} \quad 0 \leq t \leq t_b$$

where $q_0$ is the initial rate at launch and $q_b$ is the final rate at burnout. The consumption rates are constrained according to the following conditions:

$$7.5 \ \text{kg/sec} \leq q_0, \quad q_b \leq 12.5 \ \text{kg/sec}$$

The mass of propellant that is consumed at time $t$ less than or equal to $t_b$ is

$$m_p(t) = \int_0^t q \, d\tau$$

The burnout time $t_b$ may therefore be found from

$$m_p(t_b) = \text{total propellant mass}$$

Thus the instantaneous mass of the rocket is given by

$$m = \begin{cases} 
300 - m_p(t) & 0 \leq t \leq t_b \\
120 & t > t_b 
\end{cases}$$

The thrust $T$ is equal to $q v_e$, where $v_e$ is the exhaust speed of the propellant relative to the rocket and is equal to 500 m/sec, as long as the propellant lasts. Thus

$$T = \begin{cases} 
500q & 0 \leq t \leq t_b \\
0 & t > t_b 
\end{cases}$$

The drag force $D$ is given by

$$D = 0.1 \left( u^2 + v^2 \right) \frac{\rho}{\rho_s} \beta$$

The term $(\rho / \rho_s)$ is a correction factor for the atmospheric density $\rho$ at altitude $y$ relative to the density $\rho_s$ at the ground level. A model for the density ratio can be expressed as

$$\frac{\rho}{\rho_s} = (1 - \alpha y)^{4.25}$$

where $\alpha$ is equal to 0.0000225 m$^{-1}$. The quantity $\beta$ is a compressibility correction as the Mach number $M$ of the rocket becomes appreciable. The expression for $\beta$ is

$$\beta = \frac{1}{\sqrt{1 - M^2}}$$

where

$$M^2 = \frac{u^2 + v^2}{c^2}$$
in which \( c \) is the speed of sound in the atmosphere. For the altitudes that the rocket will attain, \( c \) may be taken as a constant equal to 340 m/sec.

The range is the horizontal distance that the rocket flies before returning to the ground level \((y = 0)\). The objective of the design is to choose the initial flight path angle \( \gamma_0 \) and the propellant consumption rates \( q_0 \) and \( q_b \) so that the range is maximized.

As deliverables, you need to submit the followings:
- numerical values of your optimized parameters \((\gamma_0, q_0 \text{ and } q_b)\) together with the resultant range;
- flight trajectory of the rocket;
- time history of the total mass of the rocket;
- time history of the forward thrust;
- time history of the drag force.

Your project will be graded based on
- the performance of your design and
- the accuracy of your prediction method.

This final project will be due by 5 pm on May 22, 2003. On May 22, each student will be scheduled for a 20-30 minute discussion with the instructor explaining the solution algorithm of the design project. During the last lecture, the students’ chosen design parameters will be used as input to the instructor’s code to determine the performance and accuracy of the best design. The engineer who attains
- the longest range will win Part (a) of the competition;
- the closest agreement with the instructor’s result will win Part (b) of the competition.

Warning:
Since this is a design competition, personal assistance is strictly prohibited! Any violator will automatically receive ZERO credit for this project (10% of total grade). However, you are allowed to look up any references in the literature or consult with your instructor.