OBJECTIVES

Chapter 3 - Conservation of Mass and Chemical Species Accounting

1. Define, illustrate, compare and contrast the following terms and concepts:

   Mass
   mass vs. weight
   units of measurement
   mass: \( m \) (kg, g, lbm, slug)
   amount of substance: \( n \) (kmol, gmol, lbmol, slugmol)
   (most useful when chemical reactions are involved)
   molecular weight (molecular mass): \( M \) (kg/kmol, g/gmol, lbm/lbmol)
   relationship between \( m \) and \( n \)
   density & specific volume (how are they related?)
   specific weight vs. specific gravity (how are they different?)

Application of Accounting Principle for Mass

rate of accumulation of mass within the system
amount of mass within the system: \[ m_{sys} = \iiint_r \rho \, dV \]
density vs. specific volume (\( \rho \) vs \( \nu \))
transport rate of mass across system boundaries
flow rates
mass flow rate: \[ \dot{m} = \iint_{A_c} \rho \, V_n \, dA \] (kg/s, lbm/s) where \( A_c = \) cross-sectional area
volumetric flow rate: \[ \dot{V} = \iint_{A_c} V_n \, dA \] (m³/s, ft³/s) where \( A_c = \) cross-sectional area
molar flow rate: \[ \dot{n} = \frac{\dot{m}}{M} \] (kmol/s, mol/s, lbmol/s):

local normal velocity: \( V_n \)
average velocity at a cross-section: \( V_{\text{AVG}} \)
mass and volumetric flow rate based on average velocity:\[ \dot{m} = \rho \, A_c \, V_{\text{AVG}} \] & \[ \dot{V} = A_c \, V_{\text{AVG}} \]
one-dimensional flow assumption

generation/consumption rate of mass within the system

**Empirical result ----- Mass is conserved! Its really a conservation principle!**
conservation of mass equation

rate form \[
\frac{dm_{sys}}{dt} = \sum m_{in} - \sum m_{out}
\]

finite-time form \[
m_{sys, final} - m_{sys, initial} = \sum m_{in} - \sum m_{out}
\]

Chemical Species (Compounds)

units of measurement --- same as for mass

\[ m_i = \text{mass of component } i \]

\[ n_i = \text{moles of component } i \]

mixture composition

moles of mixture: \[ n = \sum_{i=1}^{N} n_i \]

where \( N \) = number of components in the mixture

mole fractions: \[ n_f_i = \frac{n_i}{n} \]

where \( n = \) moles of mixture \[ \sum_{i=1}^{N} n_f_i = 1 \]

mass of mixture: \[ m = \sum_{i=1}^{N} m_i \]

mass (weight) fractions: \[ m_f_i = \frac{m_i}{m} \]

where \( m = \) total mass of mixture \[ \sum_{i=1}^{N} m_f_i = 1 \]

Application of Accounting Principle for Chemical Species (Compounds)

rate of accumulation of component \( i \) within the system

amount of component \( i \) within the system: \[ m_{i, sys} = \iiint V \rho_i \, dV \]

transport rate of component \( i \) across system boundaries

flow rates

mass flow rate of component \( i \): \( \dot{m}_i \) (kg/s, lbm/s, slug/s)

molar flow rate of component \( i \): \( \dot{n}_i \) (kmol/s, mol/s, lbmol/s)

generation/consumption rate of species \( i \) within the system

chemical reactions and generation/consumption

balanced reaction equations and consumption/generation terms

generation (production or creation) rate: \( \dot{m}_{i, gen} \) or \( \dot{n}_{i, gen} \)
consumption (destruction) rate: \( \dot{m}_{i, \text{con}} \) or \( \dot{n}_{i, \text{con}} \)

**chemical species accounting equation (mass basis)**

**rate form**
\[
\frac{d m_{i, \text{sys}}}{dt} = \sum m_{i, \text{in}} - \sum m_{i, \text{out}} + \dot{m}_{i, \text{gen}} - \dot{m}_{i, \text{con}}
\]

**finite-time form**
\[
m_{i, \text{sys, final}} - m_{i, \text{sys, initial}} = \sum_{i} m_{i, \text{in}} - \sum_{i} m_{i, \text{out}} + m_{i, \text{gen}} - m_{i, \text{con}}
\]

**chemical species accounting equation (mole basis)**

**rate form**
\[
\frac{d n_{i, \text{sys}}}{dt} = \sum \dot{n}_{i, \text{in}} - \sum \dot{n}_{i, \text{out}} + \dot{n}_{i, \text{gen}} - \dot{n}_{i, \text{con}}
\]

**finite-time form**
\[
n_{i, \text{sys, final}} - n_{i, \text{sys, initial}} = \sum n_{i, \text{in}} - \sum n_{i, \text{out}} + n_{i, \text{gen}} - n_{i, \text{con}}
\]

**Constitutive relation**
Examples: Ohm’s Law, Ideal Gas Model

Ideal Gas Model
universal gas constant \( R_u \) vs. specific gas constant \( R \)

1. Given one of the species accounting or conservation of mass equations and a list of assumptions, carefully indicate the consequences of each assumption. Typical assumptions include: steady-state, one-inlet/one-outlet, closed system, open system, and no chemical reactions.

2. Given one of the species accounting or conservation of mass equations, explain what each term represents physically and how it relates to the overall accounting framework discussed in Chapters 1 and 2.

3. Given one of the species accounting or conservation of mass equations, calculate the mass flow rate and volumetric flow rate at the boundary of a system.

4. Given information about the local velocity distribution and density distribution at the boundary of a system, calculate the mass flow rate and volumetric flow rate at the boundary.

5. Given a mixture composition in terms of either mass (weight) fractions or mole fractions, determine the composition in the other measure. If total mass of the mixture is specified, determine the moles or kilograms of each component in the mixture. (Best done using a simple table format.)

6. Given a problem that can be solved using conservation of mass and species accounting, you should be able to do the following tasks:
   (1) Select an appropriate system. Identify the system and its boundaries on an appropriate drawing. Describe the system and its boundaries in sufficient detail so that there is no confusion about your choice. Indicate whether the system is open or closed.
   (2) Indicate the time interval appropriate for the problem (e.g. should you use the rate-form or the finite-time form).
   (3) Clearly identify and count the number of unknowns you are trying to find. Define and use a unique symbol for each unknown.
(4) Develop a set of INDEPENDENT equations that are equal in number to the number of unknowns and are sufficient to solve for the unknowns. These equations are developed using conservation of mass, species accounting, and information given in the problem statement, e.g. physical constraints and constitutive equations.

(5) Solve for the unknown quantities.

(6) Substitute in the numerical values to find a numerical answer.

7. Given a problem with without chemical reactions where the chemical composition is uniform throughout, apply conservation of mass along with other pertinent information to solve for the unknown information.

8. Given a separation (e.g. distillation) or a mixing problem, apply conservation of mass and species accounting to solve for the unknown mass flow rates or masses and mixture compositions.

9. Given one of the following determine the other values:

    density,   specific volume,   specific weight,   specific gravity

10. Given sufficient information, use the ideal gas equation correctly to find the properties of a gas, e.g. given a pressure and a temperature, calculate the density for a specified gas assuming that it behaves as an ideal gas.