EM406
Examination III
November 7, 2003

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Show all work for credit
AND
Stay in your seat until the end of class
AND
Turn in your signed help sheet
The equations of motion for the 2-DOF system shown is
\[
\begin{bmatrix}
5 & 0 \\
0 & 1 
\end{bmatrix} \begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2 
\end{bmatrix} + \begin{bmatrix}
60 & -20 \\
-20 & 20 + k_3 
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 
\end{bmatrix} = \begin{bmatrix}
F \sin 10t \\
0 
\end{bmatrix}
\]

Determine an appropriate value of \( k_3 \) so that \( m_1 \) is stationary, that is, so that \( m_2 \) acts as a vibration absorber for \( m_1 \).
A coworker asks your help in doing a modal analysis problem. The mass matrix and modal matrix that he has determined are shown below. Answer the following questions.

\[
\phi = \begin{bmatrix}
0.425 & -0.75 & 0.263 \\
0.526 & 0 & -0.851 \\
0.425 & 0.75 & 0.263
\end{bmatrix}
\] and mass matrix \(M = \begin{bmatrix}
2 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}\)

Are these modes mass normalized (note that there will be some round-off error due to how many significant figures I kept)? If not, mass normalize the modes that are not mass normalized.

He has a second problem where he has determined that

\[
[K] = \begin{bmatrix}
4 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 2
\end{bmatrix}
\] and the modal matrix is \(\phi = \begin{bmatrix}
1 & -1 & 1 \\
3 & 0 & -1 \\
2 & 1 & wxyz
\end{bmatrix}\)

As he was writing down the modal matrix his computer crashed right before he had finished. Determine the last element of the third mode.
A three degree of freedom system is governed by the equations shown below. Determine
a) the decoupled equations of motion in terms of normal coordinates, $q_i(t)$ (write out the
decoupled equations with numbers substituted in)
b) the initial conditions for the normal coordinates.

\[
\begin{bmatrix}
2 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\ddot{x}_3
\end{bmatrix}
+ \begin{bmatrix}
3 & -2 & 0 \\
-2 & 4 & -2 \\
0 & -2 & 3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= \begin{bmatrix}
3\cos 4t \\
0 \\
0
\end{bmatrix}
\]

\[
x_1(0) = x_2(0) = x_3(0) = \dot{x}_1(0) = \dot{x}_3(0) = 0
\]
\[
\dot{x}_2(0) = 2
\]
For the double pendulum shown, the kinetic energy, potential energy and virtual work are:

\[ T = \frac{1}{2}m_1\ell_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2\left(\ell_1^2\dot{\theta}_1^2 + 2\ell_1\ell_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2) + \ell_2^2\dot{\theta}_2^2\right) \]

\[ V = -m_1g\ell_1\cos\theta_1 - m_2g(\ell_1\cos\theta_1 + \ell_2\cos\theta_2) \]

\[ \delta W = F\ell_1\cos\theta_1\delta\theta_1 + F\ell_2\cos\theta_2\delta\theta_2 \]

Using Lagrange’s equations determine the equation of motion for \( \theta_2 \).