**EM406**
Examination III
November 2, 2001

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Show all work for credit
AND
Stay in your seat until the end of class
AND
Turn in your signed help sheet
A motor is mounted on a platform that is observed to vibrate excessively at an operating speed of 6000 rpm producing a 250 N force.

a) Design a vibration absorber to add to the platform. Note that in this case, the absorber mass is only allowed to move 2 mm because of geometric and size constraints.

b) Discuss how you would find the natural frequencies of the new system. What would you need to be given?
The 3-DOF system shown is found to have the natural frequencies

\[ \omega_1^2 = 1.0 \]
\[ \omega_2^2 = 3.0 \]
\[ \omega_3^2 = 4.0 \]

and the mass normalized modal matrix:

\[
\begin{bmatrix}
0.2886 & -0.5 & 0.4083 \\
0.5774 & 0 & -0.4083 \\
0.2886 & 0.5 & 0.4083 \\
\end{bmatrix}
\]

and mass matrix \[ [M] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \]

a) Using modal analysis determine the time response of each mass if the system is given the initial conditions

\[ x_1(0) = x_2(0) = x_3(0) = \dot{x}_1(0) = \dot{x}_2(0) = 0 \quad \text{and} \quad \dot{x}_3(0) = 1 \]

b) Discuss how you would find the stiffness matrix.
The upper end of a pendulum is attached to a linear spring of stiffness $k$, where the spring is constrained to move in the vertical direction as shown. Use $y$ and $\theta$ as generalized coordinates and do not assume small angles.

a) What is the kinetic energy in terms of the generalized coordinates?
b) What is the potential energy in terms of the generalized coordinates?
c) Derive the equations of motion using Lagrange’s equations.
After doing your vibrations HW you placed it on the couch. Unfortunately, your dog ate most of your work and the only scrap of paper remaining is shown below. Is mode 1 mass normalized? Find the missing 3rd element of the second mode?

\[
[M] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
\{X\}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \{X\}_2 = \begin{bmatrix} 1 \\ 1 \\ -0.219 \end{bmatrix}
\]