Name _______________________________

EM406
Examination II
October 14, 2003

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Show all work for credit
AND
Turn in your signed help sheet
AND
Stay in your seat until the end of class
An electronic instrument is to be isolated from a panel that vibrates at frequencies ranging from 25 to 35 Hz. It is estimated that at least 80% vibration isolation is required to prevent damage to the instrument. The instrument weighs 85 N.

a) What frequency ratio is required for 80% isolation?
b) What frequency (25 Hz or 35 Hz) will be critical when finding the stiffness of the required isolator? Explain your answer. No calculations are required.
2.1) (5 pts) A system is found to have the equation of motion,

$$\ddot{y}_1 + ay_1 - y_2 = 0$$
$$\ddot{y}_2 + by_2 - y_1 = 0$$

What is the characteristic equation for this system?

2.2) (3 pts) What is a response spectrum?

2.3) (4 pts) Accelerometers give accurate measurements of the acceleration if:

a) \[ \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = 1 \]

b) \[ \frac{r}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = 1 \]

c) \[ \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = 1 \]

and if

a) \( r < 0.2 \)

b) \( r = 1 \)

b) \( r = \sqrt{2} \)

d) \( r >> 1 \)
2.4) **(4 pts)** The reading of a vibrometer becomes reliable if:

a) \[
\frac{1}{\sqrt{1-r^2}^2 + (2\zeta r)^2} = 1
\]

b) \[
\frac{r}{\sqrt{1-r^2}^2 + (2\zeta r)^2} = 1
\]

c) \[
\frac{r^2}{\sqrt{1-r^2}^2 + (2\zeta r)^2} = 1
\]

and if

a) \( r < 0.2 \)

b) \( r = 1 \)

c) \( r = \sqrt{2} \)

d) \( r >> 1 \)

2.5) **(6 pts)** The system shown below is described by \( x_G \) which is the displacement of the center of gravity measure positive up, and \( \theta \), which is the angle of the bar measured positive in a clockwise direction. When Matlab is used to obtain the natural modes of the system show below you get

\[
eval = \begin{bmatrix} 22.0 & 0 \\ 0 & 303.0 \end{bmatrix}, \quad \text{evec} = \begin{bmatrix} 2.78 & -0.030 \\ 1 & 1 \end{bmatrix}
\]

where “eval” are the eigenvalues and “evec” is the matrix of eigenvectors. Accurately sketch this system in the second mode shape. Be sure to label the relative displacements clearly.
2.6) A second order system \((m = 1 \text{ kg}, k = 10,000 \text{ N/m}, c = 2 \text{ N-s/m})\) is forced with a periodic input as shown below (only the portion of the input displacement for \(t>0\) is shown).

Determine

a) What is the fundamental frequency of the input? (2 pts)

b) Is the function odd, even or neither? What is the implication of this? (3 pts)

c) What is \(a_0\) for this function? (2 pts)

d) What changes would you need to make to your Maple worksheet? That is:
   a. What is the period of the input? (2 pts)

   b. How would you implement the \(f(t)\) in Maple? (3 pts)

   c. What is the transfer function for this problem? (3 pts)

   e) How would you determine how many terms you need to keep to characterize your steady state output? (3 pts)
A uniform bar of length, $L$, and mass, $m_2$, is connected to a cart of mass, $m_1$, as shown below. Both springs are initially undeflected. The center of gravity, $G$, is located at the center of the bar.

a) Determine the differential equations of motion for the cart and the bar assuming small angles. You must show all work for credit. Put in second order matrix form (20 pts)
If numbers are substituted into the equations found in part a) the following differential equations are obtained:

\[ \ddot{x} + 200x - 500 = 0 \]
\[ 0.2\ddot{\theta} + 36.8\theta - 50x = 0 \]

b) What are the natural frequencies and natural modes of this system? You may use Maple or Matlab, but be sure to include enough work below so I know what you did. (15 pts)
Discuss how the problem would change if the bar were pinned to the cart as shown. (5 pts)