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Show all work for credit
AND
Turn in your signed help sheet
Problem 1a
A spring has a mass of 0.3 kg and a spring constant of 40 N/m. Estimate the natural frequency of the spring.

a) 0.0866 rad/s  
b) 11.55 rad/s  
c) 20 rad/s  
d) 24.08 rad/s

Problem 1b
A second order system is excited harmonically, \( f(t) = 2 \sin(0.785t) \) and the input and total response (transient and steady state) are shown below. What is the phase angle in radians between the input and the output?
Problem 1c and 1d

Problem 1c
1a) Identify the type of damping for each system

System 1 ________________________
System 2 ________________________

How can you tell from the figures?

Problem 1d
1b) Which of the two systems has a larger natural frequency? Explain your answer.

a) System a
b) System b
c) They have the same natural frequency
d) Not enough information is given
The slender beam of mass, $M$, and length, $L$, carries two masses and is supported by a spring $k$ and a dashpot $c$. Using the angle, $\theta$, measured from the static equilibrium point as a coordinate determine:

a) the equation of motion in terms of $\theta$

b) the natural frequency in terms of the parameters given

c) the critical damping in terms of the parameters given
A 4 kg piston slides with viscous friction inside a cylinder. The upper end of an elastic spring that supports the piston moves with a harmonic motion \( x_1 = 10 \sin 12t \) where \( x \) is in millimeters. The spring constant is 400 N/m and the damping constant is 20 N-s/m. Determine:

a) the equation of motion for the mass

b) the steady state response of the mass.

c) A coworker was asked to find the range of frequencies where the amplitude is less than 11 mm and using Maple she found

\[
\{ r = -0.3248666148 \}, \{ r = -1.282365659 \}, \{ r = 1.282365659 \}, \{ r = 0.3248666148 \}
\]

Using her results, for what frequencies will the amplitude be less than 11 mm?
A 5-kg fragile glass vase is packed in chopped sponge rubber and placed in a cardboard box that has negligible weight. It is then accidentally dropped from a height of 1 m. The particular rubber exhibits a force deflection curve as shown. Neglecting damping determine:

a) an equation for the displacement of the vase as a function of time after the drop
b) the maximum deformation of the packing within the box after the fall
c) the maximum acceleration the cup experiences in g’s