Exam 3

Nov 2, 2012

| Problem 1 | ___/ 40 |
| Problem 2 | ___/ 30 |
| Problem 3 | ___/ 30 |
| **Total** | ___/ 100 |

Show all work for full credit.

Open book, computer use for computational purposes, one 8 ½ x 11” handwritten equation sheet.

If you use tabular data from your text, do **not** interpolate values. Use the nearest value in the table(s).
Problem 1 (40 pts)

A Dr. Thom Simple Cycle (DTS cycle) is a closed-system/periodic cycle consisting of only three steps as follows:

(1)→(2) Constant volume heat addition
(2)→(3) Reversible, adiabatic expansion
(3)→(1) Constant pressure heat rejection

A particular DTS cycle is to be modeled using the air standard assumptions. The air at the beginning of the heat addition is $P_1 = 90$ kPa and $T_1 = 310$ K. After the heat addition the pressure is $P_2 = 531.4$ kPa. The heat rejected in process (3)→(1) is $q_{out,31} = 456.6$ kJ/kg.

This information is summarized in the table below. (You do not need to fill in all values in the table.)

<table>
<thead>
<tr>
<th>State</th>
<th>$T$ [K]</th>
<th>$P$ [kPa]</th>
<th>$u$ [kJ/kg]</th>
<th>$h$ [kJ/kg]</th>
<th>$s$ [kJ/kg·K]</th>
<th>$P_r$</th>
<th>$p_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>310</td>
<td>90</td>
<td>221.25</td>
<td>310.24</td>
<td>1.73498</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>531.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
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</tbody>
</table>

$q_{out,31} = 456.6$ kJ/kg

(a) Sketch the $P-v$ and $T-s$ diagrams for a DTS cycle.

(b) Find the heat transfer and work per unit mass for each step in the given cycle (in kJ/kg). Assume an air-standard cycle ($R = 0.287$ kJ/kg·K.)

(c) Find the cycle efficiency.

(d) If the cycle is to produce a net work output of 400 kJ, what mass of air is required?
Problem 2 (30 pts)

Hank Hill has fired up his propane (C\textsubscript{3}H\textsubscript{8}) grill for a Texas style BBQ. (He did not invite Dr. Thom, however, as he refused to cook veggie burgers on his propane grill.) Hank’s grill is modeled as a steady state, steady flow reaction chamber as shown in the figure. Assuming that the fuel is supplied with 100\% excess air,

(a) find the balanced chemical reaction equation and  
(b) the rate of heat transfer (in kW) supplied to the BBQ.
Problem 3 (30 Points)

(a) 
   i. (3 pts) True / False  Compression ratio is given by \( r = \frac{v_{\text{max}}}{v_{\text{min}}} \)
   ii. (3 pts) True / False  Compression ratio is given by \( r = \frac{P_{\text{max}}}{P_{\text{min}}} \)

(b) (6 pts) Define adiabatic flame temperature.

(c) (10 pts) 1 k-mol of methane (CH₄) is combusted with theoretical air. The products of combustion are at a pressure of 100 kPa. The balanced reaction is given below.

\[
\text{CH}_4 + 2[O_2 + 3.76N_2] \rightarrow \text{CO}_2 + 2\text{H}_2\text{O} + 7.52\text{N}_2
\]

If the products are at \( T = 40^\circ\text{C} \), how many k-mols of water are in the liquid phase? (Assume \( 40^\circ\text{C} < T_{\text{dew}} \))
Answer only one of the next two questions.

(d) (8 pts) Prove that for a closed system undergoing a constant pressure process the heat transfer is given by

\[ Q_{12} = m(h_2 - h_1) \]

(e) (8 pts) Prove that for an isentropic process of an ideal gas with constant specific heats the pressure and temperature are related by

\[ \frac{P_2}{P_1} = \left( \frac{T_2}{T_1} \right)^{k/(k-1)} \]

(Hints: \( R = c_p - c_v \), \( k = c_p / c_v \))