Example

Dry atmospheric air is actually a mixture of gases including oxygen, nitrogen, argon and trace amounts of other gases. Consider 1 m³ of air for which the volumetric composition is 21% O₂, 78% N₂ and 1% Ar. Initially the air is at 27°C and 100 kPa. It is then heated to 227°C at constant volume.

(a) Find the apparent molar mass and the ideal gas constant for the air.
(b) Find the mass of the air.
(c) Assuming variable specific heats,
   i. find the heat transfer added to the air during the process, and
   ii. calculate the entropy generated during the process, in kJ/K.
(d) Repeat (c) by using the air tables instead of using the given mixture composition.

(a) Assume you have 1 kmole of mixture.

<table>
<thead>
<tr>
<th>i</th>
<th>( n_i ) [kmole]</th>
<th>( M_i ) [kg·kmol⁻¹]</th>
<th>( m_i = n_i \cdot M_i ) [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>O₂</td>
<td>0.21</td>
<td>32.00</td>
<td>0.72</td>
</tr>
<tr>
<td>N₂</td>
<td>0.78</td>
<td>28.01</td>
<td>21.85</td>
</tr>
<tr>
<td>Ar</td>
<td>0.01</td>
<td>39.94</td>
<td>0.3994</td>
</tr>
</tbody>
</table>

\( M_{mix} = \sum n_i \cdot M_i \)

\( R_{mix} = \frac{R \cdot \sum n_i}{M_{mix}} = \frac{8.314 \text{ kJ/} \cdot \text{kmol} \cdot \text{K}}{28.97 \text{ kg/} \cdot \text{kmol}} = 0.287 \text{ kJ/kg.K} \)

\( P \cdot V_i = mRT_i \)

\( m = \frac{P \cdot V_i}{RT} = \frac{(100 \text{ kPa})(1 \text{ m}^3)}{(0.287 \text{ kJ/kg.K})(300 \text{ K})} = 1.161 \text{ kg} \)

(c) Closed system, finite time

\( U_2 - U_1 = Q_{12} - W_{12,05} \)

\( Q_{12,IN} = m (U_2 - U_1) \)

\( O_2: \quad \overline{U}_2 - \overline{U}_1 = 10,614 - 6,242 = 4,372 \text{ kJ/} \cdot \text{kmol} \)
N₂: \[ \bar{U}_2 - \bar{U}_1 = 10,423 - 4,229 = 4,194 \text{ kJ/kmol} \]

Ar: \[ \bar{U}_2 - \bar{U}_1 = \bar{C}_v (T_2 - T_1) = 12.5 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} (500 - 300) \text{K} = 2,500 \text{ kJ/kmol} \]

\[ (\bar{U}_2 - \bar{U}_1)_{\text{mix}} = \sum y_i (\bar{U}_2 - \bar{U}_1)_i = (0.21)(4,372) + (0.78)(4,194) + (0.01)(2,500) = 4,214 \text{ kJ/kmol} \]

\[ (U_2 - U_1)_{\text{mix}} = \frac{(\bar{U}_2 - \bar{U}_1)_{\text{mix}}}{M_{\text{mix}}} = \frac{4,214 \text{ kJ/kmol}}{28.97 \frac{\text{kJ}}{\text{kmol}}} = 145.48 \frac{\text{kJ}}{\text{kg}} \]

\[ Q_{12,\text{in}} = (1.161 \text{ kg})(145.48 \frac{\text{kJ}}{\text{kg}}) = 169 \text{ kJ} \]

**Entropy Changes:**

O₂: \[ (\Delta_S - \Delta_S)_0 = \bar{S}^o (T_2) - \bar{S}^o (T_1) - R \ln \left( \frac{P_2}{P_1} \right) \]

= \[ \bar{S}^o (T_2) - \bar{S}^o (T_1) - R \ln \left( \frac{y_{o_{2}}P_2}{y_{o_{2}}P_1} \right) \]

Since constant volume:

\[ P_2 = P_1 \frac{T_2}{T_1} = 100 \text{ kPa} \frac{500 \text{ K}}{300 \text{ K}} = 166.7 \text{ kPa} \]

\[ (\Delta_S - \Delta_S)_0 = (220.589 - 205.213) - 8.314 \ln \left( \frac{5}{3} \right) = 11.129 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \]

N₂: \[ (\Delta_S - \Delta_S)_N = (206.63 - 191.682) = 10.701 \text{ kJ/kg} \]

Ar: \[ (\Delta_S - \Delta_S)_A = \bar{C}_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{U_2}{U_1} \right) \]

= \[ 12.5 \ln \left( \frac{573}{300} \right) = 6.378 \text{ kJ/kmol} \cdot \text{K} \]
\[(\bar{D}_2 - \bar{D}_1)_{mix} = \sum y_i (\bar{D}_2 - \bar{D}_1)_i = (0.21)(11.129) + (0.78)(10.701) + (0.01)(6.378)\]

\[= 10.748 \text{ kJ/kgmol} \cdot \text{k}\]

\[(\bar{D}_2 - \bar{D}_1)_{mix} = \frac{(\bar{D}_2 - \bar{D}_1)_{mix}}{M_{mix}} = ... = 0.371 \text{ kJ/kg} \cdot \text{k}\]

\[(S_2 - S_1)_{mix} = M_{mix} (\bar{D}_2 - \bar{D}_1)_{mix} = ... = 0.742 \text{ kJ/k}\]