Known: Gasoline is draining.

Find

(a) Starting with the rate form of the Cons. Mass equation and an appropriate OPEN system, determine the volumetric flow rate \( \dot{V} \) and speed of the fluid \( V \) in the connecting pipe, in m\(^3\)/min and m/s, respectively.

(b) Using Cons. Mass and an OPEN system, find the rate at which the fluid level in the lower tank changes, in m/min. Indicate whether increasing or decreasing.

(c) Repeat part (b) but instead use a CLOSED system (all of the fluid in both tanks and the connecting pipe).

Given

\( h_u \) is decreasing at a rate of 0.333 m/min

Define Variables

\( \frac{V_{gas}}{\rho_{gas}} = 0.7 \)  
\( \gamma \rightarrow \text{specific gravity} \)  
\( \rho \rightarrow \text{density} \)

\( \dot{V} = 50 \text{ m}^3/\text{min} \)
\( d_{pipe} = 0.10 \text{ m} \)
\( r_{pipe} = 0.05 \text{ m} \)
\( A_{pipe} = \text{cross-sectional area of pipe} \)
\( V_{pipe} = \text{velocity of fluid through pipe} \)
\( \dot{V}_{pipe} = \text{volumetric flow rate of fluid through pipe} \)
Analysis: "Find" suggests Cons of Mass, so begin with that. May need to integrate later.

(a) System: Oil in upper tank

Cons Mass: \[ \frac{d{m_{sys}}}{dt} = \Sigma{m_{in}} - \Sigma{m_{out}} \]

\[ \frac{d{m_{sys}}}{dt} = 0 \] b/c no mass entering system

\[ \frac{d{m_{sys}}}{dt} = \Sigma{m_{in}} - \Sigma{m_{out}} \quad (\text{Equation 1}) \]

Important plug-in relationships:

\[ {m_{sys}} = \rho_{gas} \cdot \text{Aupper} \cdot h_u \]

\[ {m_{out}} = \dot{m}_1 = \rho_{gas} \cdot A_{pipe} \cdot V_{pipe} = \rho_{gas} \cdot \dot{V}_{pipe} \]

\[ \dot{V}_{pipe} = A_{pipe} \cdot V_{pipe} \]

\[ \frac{d(\rho_{gas} \cdot \text{Aupper} \cdot h_u)}{dt} = 0 - \rho_{gas} \cdot A_{pipe} \cdot V_{pipe} \quad (\text{Equation 1 with plug-ins}) \]

\[ \rho_{gas} \cdot \text{Aupper} \cdot \frac{d(h_u)}{dt} = 0 - \rho_{gas} \cdot A_{pipe} \cdot V_{pipe} \]

*Since \( \rho_{gas} \) and \( \text{Aupper} \) are constants in this scenario, \( h_u \) is changing as the fluid leaves the tank; thus, they are moved outside the \( \frac{d}{dt} \) so becomes:

\[ \rho_{gas} \cdot \text{Aupper} \cdot \frac{d(h_u)}{dt} = 0 - \rho_{gas} \cdot A_{pipe} \cdot V_{pipe} \]

*Divide everything by \( \rho_{gas} \)

\[ \text{Aupper} \cdot \frac{d(h_u)}{dt} = 0 - A_{pipe} \cdot V_{pipe} \]

Solve for \( A_{pipe} \cdot V_{pipe} \), or \( V_{pipe} \)
\( A_{upper} \frac{d(hu)}{dt} = A_{pipe} V_{pipe} \)

\( A_{upper} \frac{d(hu)}{dt} = V_{pipe} \quad \text{negative b/c hu is decreasing} \)

\( \dot{V}_{pipe} = -(9 \text{ m}^2)(-0.333 \text{ m/min}) \)

\[ \boxed{V_{pipe} = 3 \text{ m}^3/\text{min}} \] (a)

Since \( \dot{V}_{pipe} = A_{pipe} V_{pipe} \) still need to find \( V_{pipe} \)

\( V_{pipe} = \frac{\dot{V}_{pipe}}{A_{pipe}} \)

\( A_{pipe} = \pi r_{pipe}^2 \quad \text{substitute} \)

\( V_{pipe} = \frac{\dot{V}_{pipe}}{\pi r_{pipe}^2} \)

\( V_{pipe} = \frac{3 \text{ m}^3/\text{min}}{\pi (0.05 \text{ m})^2} \)

\( V_{pipe} = 382 \text{ m/min} \Rightarrow \text{convert to m/s} \)

\[ \frac{382 \text{ m}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 6.37 \text{ m/s} = V_{pipe} \] (a)

(b) System: Oil in lower tank

\[ \text{Cons mass: } \frac{d \text{ mass}}{dt} = \Sigma \text{ in} - \Sigma \text{ out} \]
\[ \frac{dm_{\text{gas}}}{dt} = \sum m_{\text{in}} - \sum m_{\text{out}} \]

*Important relationships:*

\[ m_{\text{sys}} = \rho_{\text{gas}} A_{\text{lower}} h_e \]

\[ m_{\text{in}} = m_{\text{z}} = m_{\text{i}} \rightarrow \text{because Cons Mass on pipe} \]

**System: Pipe**

\[
\frac{dm_{\text{sys}}}{dt} = \sum m_{\text{in}} - \sum m_{\text{out}}
\]

\[ 0 \text{ because assume that amount of mass inside the pipe is constant} \]

\[ 0 = m_{\text{i}} - m_{\text{z}} \]

\[ m_{\text{z}} = m_{\text{i}} = \rho_{\text{gas}} \# \text{pipe} \]

**Equation 2 becomes:**

\[ \frac{d(\rho_{\text{gas}} A_{\text{lower}} h_e)}{dt} = m_{\text{z}} - 0 \]

\[ \rho_{\text{gas}} A_{\text{lower}} \frac{d(h_e)}{dt} = \rho_{\text{gas}} \# \text{pipe} - 0 \]

\[ A_{\text{lower}} \frac{d(h_e)}{dt} = \# \text{pipe} ightarrow \text{Solve for } \frac{d(h_e)}{dt} \]

\[ \frac{d(h_e)}{dt} = \frac{\# \text{pipe}}{A_{\text{lower}}} \]

\[ \frac{d(h_e)}{dt} = \frac{3 \text{ m}^3/\text{min}}{50 \text{ m}^2} = \frac{(l)}{0.06 \text{ m}/\text{min}} \rightarrow \text{Height is increasing} \]
(c) System: all oil in both tanks & pipe

Cons mass: \[
\frac{d\text{mass}}{dt} = \Sigma \text{in} - \Sigma \text{out}
\]

\[
\frac{d\text{mass}}{dt} = \Sigma \text{in}_0 - \Sigma \text{out}_0
\]  

(Equation 3) 

both O b/c closed system; nothing in or out

Important relationships:

\[
\text{msys} = m_{upper} + m_{pipe} + m_{lower}
\]

\[
\text{msys} = \rho_{gas} A_{upper} h_u + \rho_{gas} A_{pipe} h_{pipe} + \rho_{gas} A_{lower} h_l
\]

\[
\text{msys} = \rho_{gas} A_{upper} h_u + \rho_{gas} A_{pipe} h_{pipe} + \rho_{gas} A_{lower} h_l
\]

So Equation 3 becomes:

\[
\frac{d}{dt} (\rho_{gas} A_{upper} h_u + \rho_{gas} A_{pipe} h_{pipe} + \rho_{gas} A_{lower} h_l) = 0
\]

\[
\rho_{gas} A_{upper} \frac{d(h_u)}{dt} + 0 + \rho_{gas} A_{lower} \frac{d(h_l)}{dt} = 0
\]

Solve for \[
\frac{d(h_u)}{dt} = -\frac{A_{upper}}{A_{lower}} \frac{d(h_u)}{dt} = \frac{9 \text{ m}^2}{50 \text{ m}^2} (-0.333 \text{ m/min})
\]

\[
\frac{d(h_l)}{dt} = 0.06 \text{ m/min}
\]

It checks!!

Comments: This is a case where Cons Mass on a closed system was helpful. Also, make sure you keep your symbols/notations consistent!!
Known: Water was released from a dam

Find: Length of riverbed that absorbs water

Given:

- Width of dry riverbed, \( w = 50 \text{m} \)
- Volumetric flow rate over dam, \( \dot{V}_{\text{dam}} = 200 \text{m}^3/\text{s} \)
- Velocity of water entering ground, \( V_w = 0.01 \text{m/s} \)

Analysis:

System → Riverbed

Cons Mass: \( \frac{\text{d mass}}{\text{dt}} = \Sigma \text{in} - \Sigma \text{out} \)

Start with Cons Mass balance. It has to do with your "Find," as there is clearly a mass component to be analyzed by the system.
\[ \frac{dw_{w}}{dt} = \Delta m_{\text{in}} - \Delta m_{\text{out}} \quad (\text{Equation 1}) \]

O b/c assuming system is steady state; Why??

**Important relationships**

\[ m = \rho V = \rho A V \quad \text{sub into Equation 1} \]

\[ 0 = m_{1} - m_{2} \]

\[ 0 = \rho_{\text{water}} V_{\text{dam}} - \rho_{\text{water}} A_{\text{bed}} V_{w} \]

\[ 0 = \rho_{\text{water}} V_{\text{dam}} - \rho_{\text{water}} (L \cdot W) V_{w} \quad \text{Solve for } L \]

\[ 0 = V_{\text{dam}} - (L \cdot W) V_{w} \]

\[ (L \cdot W) V_{w} = V_{\text{dam}} \]

\[ L = \frac{V_{\text{dam}}}{W V_{w}} \]

\[ L = \frac{200 \text{ m}^3/\text{hr}}{(50 \text{ m})(0.01 \text{ m/hr})} = 400 \text{ m} \]
Known: Co-mingling fluids

Find: A set of independent Equs that can be solved for all unknowns.

Given:

![Diagram]

98% (by weight) of xylene that enters A in feed stream 1 leaves through stream 3.

96% (by weight) of benzene that enters A in feed stream 1 leaves through stream 4.

<table>
<thead>
<tr>
<th>Stream</th>
<th>Mass Flow rate (kg/h)</th>
<th>Mass fraction of Benzene</th>
<th>Mass fraction of Toluene</th>
<th>Mass fraction of Xylene</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Feed)</td>
<td>1.275</td>
<td>30.0</td>
<td>25.0</td>
<td>45.0</td>
</tr>
<tr>
<td>2 (Leaves A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 (Intermediate)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 (Leaves B)</td>
<td></td>
<td>49.0</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>5 (Leaves B)</td>
<td></td>
<td>0.0</td>
<td>1.0</td>
<td>99.0</td>
</tr>
</tbody>
</table>

* Assume Steady State system
Analysis: Begin by defining system(s) and developing equations.

System 1: \[ A \quad \uparrow m_3 \]
\[ \downarrow m_2 \]

Deal with each chemical (benzene, toluene, xylene) separately at first for each system.

So to begin, only look at benzene in system A:

\[
\textbf{Cons Mass } \frac{dw_{\text{A}}}{dt} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}}
\]

\[
\text{Benzene: } \frac{dw_{\text{A}}}{dt} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \quad (\text{Equation 1})
\]

Important relationships:

\[ w_B = \text{mass flow rate of Benzene} = \dot{m} \times \text{Mass fraction of Benzene} \]

\[ w_B = \dot{m} \times w_{f_B} = (1275 \text{ kg/h})(0.3) \]

Sub these relationships into Equation 1:

\[
\text{Benzene A: } 0 = \dot{m}_{1_B} - \dot{m}_{2_B} - \dot{m}_{3_B}
\]
\[ 0 = (1275 \text{ kg/h})(0.3) - \dot{m}_2 w_{f_B} - \dot{m}_3 w_{f_B} \quad (1)
\]

Apply same ideas to Toluene & Xylene:

Toluene A:
\[ 0 = (1275 \text{ kg/h})(0.25) - \dot{m}_2 w_{f_T} - \dot{m}_3 w_{f_T} \quad (2)
\]

Xylene A:
\[ 0 = (1275 \text{ kg/h})(0.45) - \dot{m}_2 w_{f_X} - \dot{m}_3 w_{f_X} \quad (3)
\]
Now apply Cons Mass in the same way, but use "B" as the system:

\[ \dot{m}_3 \rightarrow B \rightarrow \dot{m}_4 \]

Cons Mass \( \frac{\text{dmass}}{\text{dt}} = \sum \text{min} - \sum \text{min out} \)

\( \text{R b/c steady state} \)

Benzene B: \( \frac{\text{d}m^2}{\text{dt}} = m_3 - m_4 - m_5 \) \hspace{1cm} \text{(Equation 4)}

"Same Important Relationships" as before, so (4) becomes:

Benzene B: \( 0 = m_3 B - m_4 B - m_5 B \)
\( 0 = m_3 \text{mf}_{B3} - m_4 \text{mf}_{B4} - m_5 \text{mf}_{B5} \)
\( 0 = m_3 \text{mf}_{B3} - m_4 (0.94) - 0 \)
\( 0 = m_3 \text{mf}_{B3} - m_4 (0.99) \) \hspace{1cm} \text{(4)}

Toluene B: \( 0 = m_3 T - m_4 T - m_5 T \)
\( 0 = m_3 \text{mf}_{T3} - m_4 \text{mf}_{T4} - m_5 \text{mf}_{T5} \)
\( 0 = m_3 \text{mf}_{T3} - m_4 (0.01) - m_5 (0.01) \) \hspace{1cm} \text{(5)}

Xylene B: \( 0 = m_3 X - m_4 X - m_5 X \)
\( 0 = m_3 \text{mf}_{X3} - m_4 \text{mf}_{X4} - m_5 \text{mf}_{X5} \)
\( 0 = m_3 \text{mf}_{X3} - m_4 (0) - m_5 (0.99) \)
\( 0 = m_3 \text{mf}_{X3} - m_5 (0.99) \) \hspace{1cm} \text{(6)}
Composition Equations

We need more independent equations, so let's look at other ways to get them.

We know that Benzene, Toluene, and Xylene are the only fluids in the entire setup, so maybe we can make an equation or 2 from the mass fraction information we have about each one.

In stream 1, we see that \( m_{B1} + m_{T1} + m_{X1} = 100\% = 1 \).

Since we already know the values for the mass fractions, in stream 1, we should not use it as an equation, but we can do streams 2 & 3.

Stream 2: \( m_{B2} + m_{T2} + m_{X2} = 1 \) \hspace{1cm} (7)

Stream 3: \( m_{B3} + m_{T3} + m_{X3} = 1 \) \hspace{1cm} (8)

We still need 2 more equations /\ we have 10 unknowns.

Let's look at other information given in the problem statement.

- 98% Xylene enters A via stream 1 leaves via stream 3 so

\[ (0.98)m_B, \ m_{X1} = 91.3 \ m_{X3} \]

\[ (0.98)(1275 \text{ kg/h})(0.45) = m_2 m_{X3} \] \hspace{1cm} (9)

- 96% of Benzene enters A via stream 1 leaves via stream 4 so

\[ (0.96)m_B, \ m_{B1} = 94 \ m_{B4} \]

\[ (0.96)(1275 \text{ kg/h})(0.36) = m_{41} (0.94) \] \hspace{1cm} (10)
Now we have 10 (numbered) Equations and 10 (listed) Unknowns.
Life is Good!!

*Maple wksht used to solve is attached. Please ALWAYS include your Maple, Excel, etc to show all your work.*

Comments: When dealing w/mixture problems, be sure to deal w/each component (in this case, fluid) separately first.

Then look for alternate equations.
restart:
m[1] := 1275;
mv[B1] := 0.3;
mv[T1] := 0.25;
mv[X1] := 0.45;
mv[B4] := 0.99;
mv[T4] := 0.01;
mv[T5] := 0.01;
mv[X5] := 0.99;

mdot_1 := 1275

mdot[B1] := 0.3

mdot[T1] := 0.25

mdot[X1] := 0.45

mdot[B4] := 0.99

mdot[T4] := 0.01

mdot[T5] := 0.01

mdot[X5] := 0.99

> e1 := 0 = (1275 * 0.3) - (mdot[2] * mv[B2]) - (mdot[3] * mv[B3]);
> e2 := 0 = (1275 * 0.25) - (mdot[2] * mv[T2]) - (mdot[3] * mv[T3]);
> e3 := 0 = (1275 * 0.45) - (mdot[2] * mv[X2]) - (mdot[3] * mv[X3]);
> e4 := 0 = (mdot[3] * mv[B3]) - (mdot[4] * 0.99);
> e5 := 0 = (mdot[3] * mv[T3]) - (mdot[4] * 0.01) - (mdot[5] * 0.01);
> e6 := 0 = (mdot[3] * mv[X3]) - (mdot[5] * 0.99);
> e7 := 1 = mv[B2] + mv[T2] + mv[X2];
> e8 := 1 = mv[B3] + mv[T3] + mv[X3];
> e9 := 0 = (0.98 * m[1] * mv[X1]) - (mdot[3] * mv[X3]);
> e10 := 0 = (0.96 * m[1] * mv[B1]) - (mdot[4] * mv[B4]);

> e5 := 0 = m[3] * mv[T3] - 0.01 * m[4] - 0.01 * m[5]
> e8 := 1 = mv[B3] + mv[T3] + mv[X3]
> e9 := 0 = 562.2750 - m[3] * mv[X3]
> e10 := 0 = 367.200 - 0.99 * m[4]
Solution := 
  solve({e1, e2, e3, e4, e5, e6, e7, e8, e9, e10}, {mdot[2], mdot[3], mdot[4], mdot[5], mf[B2], mf[B3], mf[T2], mf[T3], mf[X2], mf[X3]});

Solution := {mdot_5 = 567.9545455, mdot_4 = 370.9090909, mdot_3 = 938.8636364,
              mdot_2 = 336.1363636, mf_{X3} = 0.5988888889, mf_{X2} = 0.03413793103, mf_{T3} = 0.01000000000,
              mf_{T2} = 0.9203448276, mf_{B3} = 0.3911111111, mf_{B2} = 0.04551724138}
**Known:** Rocket sled slowed by air drag

**Find:**
(a) Value of \( k \) if \( F_{\text{drag}} = kV^2 \)
(b) Time to slow to 700 ft/s after engine shuts off. And distance travelled.

**Given:**
- \( W_{\text{sled}} = 3220 \text{ lb} \)
- \( V_i = 700 \text{ ft/s} \)
- Mass Velocity
- \( F_T = 3000 \text{ lb} \)
- Thrust

**Analysis:**

Strategy: Try linear momentum since forces involved.

System → Sled Closed
Time → Finite
Count → \( LM \)

\[
\frac{dP_x}{dt} = -F_{\text{drag}} + F_T
\]

\[
\Rightarrow \quad \frac{dV_x}{dt} = -kV^2 + F_T
\]

At constant \( V = 700 \text{ ft/s} \)

\[
\frac{dV_x}{dt} = 0 \quad \Rightarrow \quad k = \frac{F_T}{V_{ss}^2} = \frac{3000 \text{ lb}}{(700 \text{ ft/s})^2} = 16.33 \times 10^{-3} \frac{\text{lb} \cdot \text{s}^2}{\text{ft}^2}
\]
Now in general
\[ m_s \frac{dV_x}{dt} = -k \sqrt{V} + F_T \]

If \( F_T = 0 \) suddenly (engine cuts out)
when \( V = V_{ss} \)
\[ m_s \frac{dV_x}{dt} = -k \sqrt{V} \]
\[ \int \frac{dV_x}{V_x^2} = \int \frac{-k}{m_s} dt \]
\[ \frac{1}{V_x} = -\frac{k}{m_s} t + C \]

At \( t = 0 \), \( V_x = V_{ss} = 700 \text{ mph} \quad \Rightarrow \quad C = -\frac{1}{V_{ss}} \]

Thus
\[ \frac{1}{V_{ss}} - \frac{1}{V_x} = -\frac{k}{m_s} t \]
\[ 1 - \frac{V_{ss}}{V_x} = -\frac{k V_{ss}}{m_s} t \]
\[ \frac{V_x}{V_{ss}} = \frac{1}{1 + \frac{k V_{ss}}{m_s} t} \]

Now
\[ K \frac{V_{ss}}{m_s} = 16.33 \times 10^{-3} \frac{\text{mph} \cdot \text{s}^2}{\text{ft}^2} \times \frac{700 \text{ mph}}{5} \left( \frac{3220 \text{ lb}}{32174 \text{ ft/s}^2} \right) \]
\[ = 0.1142 \text{ s}^{-1} = \frac{1}{8.7552 s} \]
Solving for $t$ for $V = 700 \text{ ft/s} \rightarrow 70 \frac{\text{ft}}{\text{s}}$

$$t = \frac{\sqrt{\frac{V_s}{V}} - 1}{k \frac{V_s}{m_s}} = \frac{\frac{700}{70} - 1}{(1/8.7552) \text{s}}$$

$$= 78.8 \leq [\text{CSS}]$$

See distance calculation on page 4

**Comment**

- Careful with units
- Notice how $k \frac{V_s}{m_s} = \frac{1}{\tau_c}$

where $\tau_c$ is a characteristic time.

Then

$$\frac{V}{V_{ss}} = \frac{1}{1 + \frac{t}{\tau_c}}$$

- Also since $k = \frac{F_r}{V_s^2}$

$$k \frac{V_s}{m_s} = \frac{F_r}{V_{ss}^2} \frac{V_s}{(W/d)} = \frac{F_r}{W} \frac{d}{V_{ss}}$$

$$\therefore \tau_c = \frac{W}{F_r} \frac{V_{ss}}{d}$$
Solving for distance travelled

\[
V = \frac{dx}{dt} = \frac{V_{ss}}{1 + t/t_c}
\]

Separate variables

\[
\int dx = V_{ss} \int \frac{2c dt}{2c + t}
\]

Indefinite integral

\[
x = V_{ss} \ln \left( \frac{t + 2c}{2c} \right) + C
\]

At \( t = 0 \), \( x = 0 \)

\[
\therefore C = -2c \ln(2c) V_{ss}
\]

So the solution is

\[
x = \left[ 2c \ln \left( \frac{t + 2c}{2c} \right) - 2c \ln(2c) \right] V_{ss}
\]

\[
x = 2c V_{ss} \ln \left( \frac{t + 2c}{2c} \right)
\]

\[
c = 8.7552 \text{ s} \quad V_{ss} = 700 \text{ ft/s}
\]

For \( t = 78.8 \text{ s} \)

\[
x = (8.7552 \text{ s}) \left( 700 \text{ ft/s} \right) \ln \left[ \frac{78.8}{8.7552} + 1 \right]
\]

\[
= 14.11 \times 10^3 \text{ ft}
\]
Known: Mailbox hit by a stream of water

Find: (a) Shear force of post on mailbox
     (b) Force of water acting on mailbox

Given: \( P_{tm} = 150 \text{ kPa} \)

\( A = 300 \text{ mm}^2 \)

\( V = 25 \text{ m/s} \)

Analysis:

(a) System:

Apply L.M.

\[ \frac{dH_k}{dz} = F_{pm} + R_x + m_1 \dot{V}_x \]

\( X \) Uniform all around so rotation pressure forces cancel

\[ R_x = -m_1 \dot{V}_x \]

\[ m_1 = \rho AV = \left(1000 \frac{\text{kg}}{\text{m}^3}\right) \left(300 \times 10^{-6} \text{ m}^2\right) \left(25 \text{ m}\right) \]

\[ = 7.5 \text{ kg/s} \]

\[ R_x = -\left(7.5 \frac{\text{kg/s}}{5} \right) \left(25 \frac{\text{m}}{5} \right) = -187.5 \frac{\text{kg} \cdot \text{m}}{5^2} \]

\[ = -187.5 \text{ N} \]
(b) Force of water on mailbox —

\[ \frac{dP_x}{dty} = F_w + R_x - \text{Palm A} \]

\[ \Rightarrow F_w = -R_x + \text{Palm A} \]

\[ F_w = 187.5 \, \text{N} + \text{Palm A} \]

\[ \text{Comment} \]

Water-mailbox contact

At contact surface the water exerts a force on the mailbox.
Known: Block acted on by a time varying force

Find: (a) Time $t_1$ when block first moves
(b) Max velocity of block
(c) Time $t_3$ when block stops moving

Given:
- $m_{\text{block}} = 125 \text{ lbm}$
- $F_0 = 100 \text{ lb}$

Analysis:

Strategy → Apply Cons. of LM to block

\[
\begin{align*}
\sum F &= mg - F_N \\
\sum F_x &= P - F_N = P - \mu mg \\
\end{align*}
\]

\[
\begin{align*}
\frac{dV_x}{dt} &= P - \mu mg \\
\end{align*}
\]

(a) At point of incipient motion $\frac{dV_x}{dt} = 0, \ V_x = 0$ \[m \frac{dV_x}{dt} = P - \mu mg\]

\[\therefore \ P = \mu_s mg = (0.5)(125 \text{ lb})\]
\[ P_f = 62.5 \text{ ft} \quad \rightarrow \quad t_1 = \frac{62.5}{100} (8 \text{ s}) \]
\[ t_1 = 5 \text{ s} \]

(b) \[ \frac{dV_x}{dt} = 0 \quad \Rightarrow \quad V = \text{const} \]

\[ m \frac{dV_x}{dt} = P - m \cdot g \]
\[ P = m \cdot g = (0.40 \text{ lb}) (125 \text{ lb}) = 50 \text{ lb} \]

\[ t_2 = 8 \text{ sec} + \frac{100 - 50}{100} (8 \text{ s}) \]
\[ = 12 \text{ sec} \]

\[
\begin{cases}
0 \leq t \leq 8 \text{ s} & P = \left(\frac{t}{8}\right) 100 \text{ lb}
\\
8 \leq t \leq 16 \text{ s} & P = 200 \text{ lb} - 100 \text{ lb} \left(\frac{t}{8}\right)
\\
t \geq 16 \text{ s} & P = 0
\end{cases}
\]

Calculate \( V_x \) vs \( t \)

\[ \frac{dV_x}{dt} = \frac{P}{m} - mg = \frac{P}{mg} - mg \]

\[ \frac{dV_x}{dt} = \frac{P}{mg} - mg \]

For \( 8 \leq t \leq 16 \)
\[ V_t = \int_{55}^{t} \frac{P}{W} g \, dt - \left[ \frac{P}{W} g (t - 55) \right] \]

\[ = \int_{55}^{85} \frac{P}{W} g \, dt + \int_{85}^{t} \frac{P}{W} g \, dt + \left[ \frac{P}{W} g (t - 55) \right] \]

\[ \kappa = \int_{55}^{85} \left( \frac{t}{85} \right) \left( \frac{100}{125} \right) \left( 32.174 \frac{rt}{s^2} \right) \, dt = \left( 3.2174 \frac{rt}{s^2} \right) \left( \frac{t^2}{2} \right) \bigg|_{55}^{85} \]

\[ = 62.74 \frac{rt}{s} \]

\[ \kappa \kappa = \int_{85}^{t} \left[ \frac{200}{125} - \frac{100 (1/5)}{125 (85)} \right] g \, dt \]

\[ = \int_{85}^{t} \left( 51.48 \frac{rt}{s^2} \right) \, dt - \int_{85}^{t} \left( 3.2174 \frac{rt}{s^2} \right) t \, dt \]

\[ = \left( 51.48 \frac{rt}{s^2} \right) (t - 85) - \left( 3.2174 \frac{rt}{s^2} \right) \left( \frac{t^2}{2} \right) \bigg|_{85}^{t} \]

\[ = \left( 51.48 \frac{rt}{s^2} \right) (t - 85) - \left( 3.2174 \frac{rt}{s^2} \right) \left( \frac{t^2}{2} - 325^2 \right) \]

\[ = \left( 51.48 \frac{rt}{s^2} \right) t - 1.6087 \frac{rt}{s^2} \left[ t^2 - 308.88 \frac{rt}{s} \right] \]

\[ \therefore \int_{55}^{t} \frac{P}{W} g \, dt = 51.48 \frac{rt}{s^2} t - 1.6087 \frac{rt}{s^2} t^2 - 246.44 \frac{rt}{s} \]
\[ M_x g (t-55) = (0.4)(32.184 \frac{m}{s^2}) (t-55) \]
\[ = 12.87 \frac{m}{s^2} t - 64.35 \frac{m}{s} \]

Finally,
\[ V_x = \int_{55}^{t} \frac{P}{W} g dt - M_x g (t-55) \]
\[ V_x = (57.48 - 12.87) \frac{m}{s^2} t - 16087 \frac{m}{s^3} t^2 \]
\[ - (246.14 - 64.35) \frac{m}{s} \]
\[ V_x = (38.61 \frac{m}{s}) t - (16087 \frac{m}{s^3}) t^2 - 181.74 \frac{m}{s} \]

At \( t = 12 \text{ s} \),
\[ V_x = 49.88 \frac{m}{s} \]

(c) Time to stop moving —

Check at \( t = 16 \text{ s} \),
\[ V_{x/16} = 24.14 \frac{m}{s} \]

So still moving at \( t = 16 \text{ s} \)
For $t > 16s \quad P = 0$

\[
\frac{dV}{dt} = -mg
\]

\[
V_x - V_{x(16s)} = -mg (t - 16s)
\]

\[
\therefore t_3 - 16s = \frac{0 - 24.14 \frac{m}{s}}{(0.4)(32.174 \frac{m}{s^2})} = 1.88s
\]

\[
t_3 = (16 + 1.88) s = 17.88s
\]
Known:  TWO SWIMMERS DIVE OFF A BOAT
Find:  a) BOAT'S FINAL V IF BOTH SWIMMERS DIVE TOGETHER
       b) " " " " " A DIVES FIRST
Given:

Two swimmers $A$ and $B$, of mass 75 kg and 50 kg, respectively, dive off the end of a 200-kg boat. Each swimmer has a relative horizontal velocity of 3 m/s when leaving the boat.

- BOAT INITIALLY @ REST.

Analysis.

a) SYSTEM: BOTH SWIMMERS ≠ B: BEFORE & AFTER DIVE

PROP: LM
BME: FINITE

\[
\begin{align*}
&\text{CoLM} \rightarrow \text{ x DIR:} \\
&\frac{d}{dt} \left( P_{x,\text{sys}} \right) = \sum F_x^{\text{ext}} + \sum m_i \dot{V}_i - \sum \dot{m} V_i \\
&\int_{P_{x,\text{sys}}}^{P_{x,\text{sys}}} dt = \int_0^t dt \quad \text{CLOSED SYSTEM} \\
&\int_{P_{x,\text{sys}}}^{P_{x,\text{sys}}} dt = 0 \quad \text{INTEGRATE TO GET FINITE TIME} \\
&P_{x,2} - P_{x,1} = 0 \\
&m_B (-V_{BT,2}) + m_A V_{S,2} + m_B V_{S,2} = 0 \\
&\begin{bmatrix}
& P_{x,2} \\
& m_B \\
& m_B T
\end{bmatrix} = 0 \\
V_{BT,2} = \frac{(m_A + m_B) V_{S,2}}{m_B T} \quad [1]
\end{align*}
\]
NOW WE NEED TO BE CAREFUL ABOUT RELATIVE VELOCITIES HERE.
ALL VEL IN [1] ARE TAKEN W.R.T. GROUND. WE AREN'T GIVEN
$V_{s,2}$, THEN, BUT RATHER $V_{s,2}/V_{t,2}$ IN VECTOR FORM.

$$
\overrightarrow{V_{s,2}} = \overrightarrow{V_{t,2}} + \overrightarrow{V_{s,2}/V_{t,2}}
$$

**THE X-COMP. OF THIS IS**

$$
V_{s,2} = - V_{t,2} + V_{s,2}/V_{t,2} \tag{2}
$$

[1] BECOMES

$$
V_{t,2} = \frac{(m_A + m_B)(-V_{t,2} + V_{s,2}/V_{t,2})}{m_{t,2}}
$$

$$
V_{t,2} = \frac{(m_A + m_B)(V_{s,2}/V_{t,2})}{m_{t,2} + m_A + m_B} = \frac{(75 + 50)(3 \text{ m/s})}{200 \text{ kg} + 75 \text{ kg} + 50 \text{ kg}}
$$

$$
= \frac{1.15 \text{ m/s}}
$$

b) **SYS: BOTH SWIMMERS & BOAT BEFORE & AFTER DIVE:**

**COLM (1) → (2) ONLY:** *(X-DIR)*

$$
\frac{d}{dt} (P_{x,sys}) = \sum F_x = \sum P_{x,i} - \sum P_{x,0}
$$

$$
\frac{d}{dt} P_{x,sys} = 0
$$

$$
P_{x,sys,2} - P_{x,sys,1} = 0
$$
\[
\left[ \frac{m_{BT} + m_B V_{BT,2} + m_A V_{A,2}}{P_{x,2}} - 0 \right] = 0 \quad (3)
\]

\text{SIMILAR TO PART a) WE HAVE}

\[V_{A,2} = -V_{BT,2} + \sqrt{\frac{V_{A,2}/B_{T,2}}{3 \text{ m/s}}}\]

\text{BECOMES}

\[
(m_{BT} + m_B)(-V_{BT,2}) + m_A (-V_{BT,2} + V_{A,2}/B_{T,2}) = 0
\]

\[
V_{BT,2} = \frac{m_A V_{A,2}/B_{T,2}}{m_{BT} + m_B + m_A} = \frac{(75)(3)}{200 + 50 + 75} = 0.692 \text{ m/s}
\]

\text{NOW LET’S LOOK AT COLL IN X-DIR FOR (2) TO (3)}

\[
\frac{dP_{x,3}}{dt} = \dot{\theta} + \ddot{\theta} - \dot{\theta}_0
\]

\[
P_{x,3} - P_{x,2} = 0 \quad V_{A,3} = V_{x,2}
\]

\[
\left[ m_{BT}(-V_{BT,3}) + m_A V_{A,3} + m_B V_{B,3} \right] - \left[ (m_{BT} + m_B) V_{BT,2} + m_A V_{A,2} \right] = 0
\]

\[m_{BT}(V_{BT,3}) + m_B V_{B,3} - (m_{BT} + m_B)(-V_{BT,2}) = 0
\]

\text{AGAIN:}

\[V_{B,3} = -V_{BT,3} + \sqrt{\frac{V_{B,3}/B_{T,3}}{3 \text{ m/s}}}\]

\[
m_{BT}(-V_{BT,3}) + m_B \left[ V_{BT,3} + V_{B,3}/B_{T,3} \right] - (m_{BT} + m_B)(-V_{BT,2}) = 0
\]
\[ V_{BT,3} = \frac{(m_{BT} + m_B)(V_{BT,2}) + m_B \cdot V_{B,3/VT3}}{m_B + m_{BT}} \] (5)

\[ = \frac{(200 + 50)(0.692) + (50)(3)}{(50 + 200)} = 1.292 \text{ m/s} \]

C) REPEATING ANALYSIS W/ B DIVING FIRST LOOKS JUST LIKE PART b) W/ A \& B REVERSED.

4) BECOMES:

\[ V_{BT,2} = \frac{m_B \cdot V_{B,2/VT}}{m_{BT} + m_B + m_A} = \frac{(50)(3)}{200 + 50 + 75} = 0.462 \text{ m/s} \]

5) BECOMES

\[ V_{BT,3} = \frac{(m_{BT} + m_A) V_{BT,2} + m_A V_{A,3/VT3}}{m_A + m_{BT}} \]

\[ = \frac{(200 + 75)(0.462) + (75)(3)}{75 + 200} = 1.280 \text{ m/s} \]
Known: Mailbox hit by stream of water

Find: (a) Shear force \( \tau \) of post on mailbox
(b) Force of water acting on mailbox

Given:

\[ P_{\text{atm}} = 100 \text{ kPa} \]
\[ A = 300 \text{ mm}^2 \]
\[ V = 25 \text{ m/s} \]

Analysis:

(a) System: mailbox & some water

Use free body diagram

\[ \frac{dP_{\text{sys}}}{dt} = \sum F_{\text{ext}} + \sum m_{\text{in}} \vec{V}_{\text{in}} - \sum m_{\text{out}} \vec{V}_{\text{out}} \]

for \( x \)-direction

\[ \frac{dP_{\text{sys}}}{dt} = F_{\text{atm}} + R_x + m_{\text{in}} \vec{V} \]

so uniform flow around, so atm force cancel

\[ \sum = R_x + m_{\text{in}} \vec{V} \]

\[ R_x = - m_{\text{in}} \vec{V} \]

Remember: \( \vec{m} = \rho AV \) so

\[ R_x = - \rho AV \cdot \vec{V} \]
\[ R_x = -\left(\frac{1000 \text{ kg}}{\text{m}^3}\right) \left(300 \text{ mm}^2\right) \left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)^2 \left(25 \text{ m/s}\right) \left(25 \text{ m/s}\right) \]

\[ R_x = -187.5 \text{ kg} \cdot \text{m/s}^2 \]

(b) Force of water on mailbox

\[ F_W = \begin{cases} mg & \text{if } \text{no } P_{\text{atm}} \text{ here} \\ -P_{\text{atm}} A & \text{only concerned w/x-direction} \end{cases} \]

Apply conservation of momentum in x-direction

\[ \frac{dH_{\text{sys}}}{dt} = \sum F_{\text{ext}} + \sum \dot{m} \vec{v}_{\text{in}} - \sum \dot{m} \vec{v}_{\text{out}} \]

\[ 0 = F_W + R_x - P_{\text{atm}} A \]

\[ F_W = P_{\text{atm}} A - R_x \]

\[ F_W = \left(100 \times 10^3 \text{ N/m}^2\right) \left(300 \text{ mm}^2\right) \left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)^2 + 187.5 \text{ N} \]

\[ F_W = 30 \text{ kN} \]
Known: block acted on by time-varying force
Find: (a) Time t1, when block first moves
      (b) Max velocity of block
      (c) Time t3 when block stops moving

Given.

\[ P = 100 \text{ lbf} \]
\[ M_{\text{block}} = 125 \text{ lbm} \]
\[ \mu_s = 0.5 \]
\[ \mu_k = 0.41 \]

Analysis: System = block

Apply cons in Mom

\[ F_f \]
\[ F_n \]

\[ F_{\text{ext}} \]
\[ \Sigma \text{in} \]
\[ \Sigma \text{out} \]

\[ \frac{dP_{\text{sys}}}{dt} + \Sigma F_{\text{ext}} + \Sigma \text{in} \dot{V}_{\text{in}} - \Sigma \text{out} \dot{V}_{\text{out}} \]

\[ \dot{V}_y = 0 \]
\[ \text{no mass entering/leaving} \]

\[ \text{so} \]

\[ F_n = mg \]
Cons. Lin Mom x direction

\[ \frac{dP}{dt} = P - F_{\text{friction}} \]

Remember: \( F_{\text{friction}} \) (hereafter referred to as \( F_f \)) = \( \mu F_n \)

\[ \frac{dP}{dt} = P - \mu F_n \]

Also remember: \( P = mv \) and \( F_n = mg \) (From above)

so

\[ \frac{dv}{dt} = P - \mu mg \]

\[ m \frac{dv}{dt} = P - \mu mg \]

(a) at point of incipient motion \( \frac{dv}{dt} = 0 \), \( v = 0 \), and \( \mu = \mu_s \)

so

\[ m \frac{dv}{dt} = P - \mu mg \]

\[ 0 = P - \mu mg \]

\[ P = \mu mg = (0.5)(125 \text{ lbf}) \]

\[ P = 62.5 \text{ lbf} \] at \( t_1 \)

To find \( t_1 \), set up ratio: \( \frac{62.5 \text{ lbf}}{t_1} = \frac{100 \text{ lbf}}{5 \text{ s}} \)

\[ t_1 = 5 \text{ s} \]
(b) at max velocity \((v_{\text{max}})\) \(\Rightarrow \frac{dv_x}{dt} = 0\) and \(\mu = \mu_k\)

\[m \frac{dv_x}{dt} = P - \mu_k m g\]

\[P = \mu_k m g = (0.4)(125 \text{ lbf})\]

\[P = 50 \text{ lbf}\]

\[t_2 = 8 \text{ sec} + \left(\frac{100 - 50 \text{ lbf}}{100 \text{ lbf}}\right) 8 \text{ sec}\]

\[t_2 = 12 \text{ sec}\]

\[\text{FYI}\]

\[\begin{cases} 
0 \leq t < 8 & P = (\frac{4}{8}) 100 \text{ lbf} \\
8 \leq t \leq 16 & P = 200 \text{ lbf} - 100 \text{ lbf}(\frac{4}{8}) \\
+16 & P = 0
\end{cases}\]

Calculate \(v_x v_s t\)

\[m \frac{dv_x}{dt} = P - \mu_k m g\]

\[\frac{dv_x}{dt} = \frac{P}{m} - \mu_k g = \frac{P}{m/g} - \mu_k g\]

\[\frac{dv_x}{dt} = \frac{P}{mg} - \mu_k g\]
\[
\int dV_x = \int \frac{P}{W} g dt - \mu_k g
\]

\[
V_x = \int_0^t \frac{P}{W} g dt - \mu_k g(t - 5)
\]

\[
v = \int_5^8 \frac{P}{W} g dt + \int_8^t \frac{P}{W} g dt
\]

\[
* = \left[ \int_5^8 \left( \frac{t}{8s} \right) \left( \frac{100}{125} \right) (32.174 \text{ ft/s}^2) \right]_{s_5}^{s_8} = (3.2174 \text{ ft/s}^2) \frac{12}{2}
\]

\[
= 19.214 \text{ ft/s}
\]

\[
* \times \times = \int_8^t \left[ \frac{200}{125} - \frac{100}{125} \left( \frac{t}{8s} \right) \right] g dt
\]

\[
= \int_8^t (1.418 \text{ ft/s}^2) dt - \int_8^t (3.2174 \text{ ft/s}^3) \frac{t^2}{2} dt
\]

\[
= (1.418 \text{ ft/s}^2)(t - 8s) - (3.2174 \text{ ft/s}^3) \frac{t^2}{2} \bigg|_8^t
\]

\[
= (1.418 \text{ ft/s}^2)(t - 8s) - (3.2174 \text{ ft/s}^3) \left( \frac{t^2}{2} - 32 s^2 \right)
\]

\[
\int_5^8 \frac{P}{W} g dt = (1.418 \text{ ft/s}^2) t - (1.6067 \text{ ft/s}^3) t^2 - 24(0.14 \text{ ft/s})
\]
\[ \mu_k g (t - 50) = (0.4) (32.174 \, \text{ft/s}^2)(t - 50) \]
\[ = (12.87 \, \text{ft/s}) t - 64.35 \, \text{ft/s} \]

Plug everything back in: \( t = 12 \) s

\[ V_x = (38.61 \, \text{ft/s}) + (1.6087 \, \text{ft/s}^2) t^2 - 18.17 \, \text{ft/s} \]

At \( t = 12 \, \text{s} \)

\[ V_x = 119.88 \, \text{ft/s} \]

\[ t = 12 \, \text{s} \]

\[ V_{\text{max}} \]

\( V_{\text{max}} \) is the maximum velocity.

Stop running... check \( t = 11 \, \text{s} \)

\[ V_{x,t = 11 \, \text{s}} = 24.14 \, \text{ft/s} \]

so still moving.

for \( t > 11 \, \text{s} \)

\[ P = 0 \]

\[ \frac{dV_x}{dt} = -\mu_k g \]

\[ V_x = V_{x,t = 11 \, \text{s}} = -\mu_k g (t - 11 \, \text{s}) \]

\[ t - 11 \, \text{s} = \frac{0 - 24.14 \, \text{ft/s}}{(0.4)(32.174 \, \text{ft/s}^2)} \]

\[ = 1.88 \, \text{s} \]

\[ t = 11 \, \text{s} + 1.88 \, \text{s} \]

\[ = 17.88 \, \text{s} \]
Known: 2 swimmers dive off a boat
Find: (a) \( V_{\text{boat}} \) if both swimmers dive together
(b) " " " " A dives 1st

Given: 2 swimmers, A & B, dive off the end of a 200 kg boat. Each has relative horizontal velocity 3 m/s when leaving.

Boat initially @ rest

\[ m_A = 75 \text{ Kg} \]
\[ m_B = 50 \text{ Kg} \]
\[ m_{\text{Boat}} = 200 \text{ Kg} \]

(a) System: both swimmers & boat before & after dive
Apply Cons Lin Mom Finite

\[ \sum F \text{ & } \sum M = 0 \]
\[ m_A V_{\text{BTZ}} + m_B V_{\text{BTZ}} = m_{\text{Boat}} V_{\text{BTZ}} \]

Cons Lin Mom - x-direction

\[ \frac{dP_{\text{sys,x}}}{dt} = \sum F_x \]
\[ = m_A V_{\text{BTZ}} + m_B V_{\text{BTZ}} - m_{\text{Boat}} V_{\text{BTZ}} \]

Closed

So integrate to get finite time
\[ \int_{P_{\text{sys},x}}^{P_{x}} \frac{dP}{dt} = \int_{t_0}^{t} 0 \ dt \]

\[ P_{x_2} - P_{x_1} = 0 \]

\[ \frac{m_A V_{S_2} + m_B V_{S_B}}{V_{S_2}} = 0 = \frac{m_B \text{boat}}{m_{\text{boat}}} \]

\[ V_{BT_2} = \frac{(m_A + m_B) V_{S_2}}{m_{\text{boat}}} \quad (1) \]

*Must be careful about relative velocities!! All velocities in (1) are given w/raspct to GROUND. We are not given \( V_{S_2} \), rather we have \( V_{S_2}/BT_2 \) in vector form.

\[ \overrightarrow{V_{S_2}} = \overrightarrow{V_{BT_2}} + \overrightarrow{V_{S_2}/BT_2} \]

The \( x \)-component of this is

\[ V_{S_2} = -V_{BT_2} + V_{S/Z}/BT_2 \quad (2) \]

This is 3 m/s
(4) \[ V_{BT2} = \frac{(m_A + m_B)(-V_{BT2} + \frac{v_{A2}/v_{BT2}}{m_{Boat}})}{m_{Boat}} = 1.15 \text{ m/s} \]

(b) System: both swimmers & boat before & after dive

\[ V_{x1} \quad \Rightarrow \quad +x \]

(1)

\[ \begin{array}{c}
\text{Cons lin Mom. } x \text{-dir only} \\
\dot{P}_{x_{sys}} = \sum F \quad \Rightarrow \quad \text{ma}_{\text{lin}} V_{in} - \text{Enjout Vout} \\
\text{no m. flow} \\
\frac{d\dot{P}_{x_{sys}}}{dt} = 0 \\
\dot{P}_{x2} = \dot{P}_{x1} = 0 \\
(m_{mB} + m_B)(-V_{BT2}) + m_A V_{A2} - 0 = 0 \\
\frac{d\dot{P}_{x}}{dt} = 0 \\
\]
Similar to Part (a)\textsuperscript{\textcircled{2}}.

\[ V_{A_{12}} = -V_{B_{T_{12}}} + \overbrace{V_{A_{2/B_{T_{12}}}}} - 3 \text{ m/s} \]

(3) Becomes \textsuperscript{\textcircled{2}}.

\[ (m_{B_T} + m_B)(-V_{B_{T_{12}}}) + m_A (-V_{B_{T_{12}}} + V_{A_{2/B_{T_{12}}}}) = 0 \]

\[ V_{B_{T_{12}}} = 0 \text{ m/s} \]

Now Cons Lin Mom in x-direction from 2 to 3.

\[ \frac{d\overrightarrow{P_{sys,x}}}{dt} = \overrightarrow{\Delta P_{x_1}} + \overrightarrow{\Delta P_{x_2}} - \overrightarrow{\Delta P_{x_3}} \]

\[ P_{x_3} - P_{x_2} = 0 \]

\[ [m_{B_{T_{3}}} (-V_{B_{T_{3}}}) + m_A (\sqrt{V_{A_{3}} + m_B V_{B_{3}}}) - \left( (m_{B_T} + m_B)(-V_{B_{T_{12}}}) + m_A V_{A_2} \right] = 0 \]

\[ P_{x_3} \quad P_{x_2} \]

\[ m_{B_T} (-V_{B_{T_{3}}}) + m_B V_{B_{3}} - (m_{B_T} + m_B)(-V_{B_{T_{12}}}) = 0 \]

Again \textsuperscript{\textcircled{2}}.

\[ V_{B_{3/2}} = -V_{B_{T_{3}}} + V_{B_{3/B_{T_{3}}}} - 3 \text{ m/s} \]
\[ m_{BT} (-V_{B13}) + m_B \left[ V_{BT3} + V_{B3/BT3} \right] - [m_{BT} + m_B] - V_{BT2} = 0 \]

\[ V_{BT2} = 1.292 \text{ m/s} \]
Known Pulley mass system

Find (a) Velocity of each mass, LM, & AM
   (b) Angular momentum about P if pulleys are locked

Given

\[ w = 2 \text{ rad/s} \]

\[ D = 0.5 \text{ m} \]
\[ d = 0.25 \text{ m} \]
\[ m_A = 10 \text{ kg} \]
\[ m_B = 5 \text{ kg} \]

Analysis

(a) Velocities...remember the relationship \( v = w \cdot r \)

\[ v_A = \frac{d}{2} \cdot w = \left(0.25 \text{ m}\right) \left(2 \text{ rad/s}\right) = 0.25 \text{ m/s} \uparrow \]

\[ v_B = \frac{D}{2} \cdot w = \left(0.5 \text{ m}\right) \left(2 \text{ rad/s}\right) = 0.5 \text{ m/s} \downarrow \]

* Be sure to specify direction 😊
Angular Momentum about "P"

Find magnitude & direction for each mass (A & B).
Remember $I = m\vec{r} \times \vec{v}$

$I_{PA} = m_A(\vec{r}_A \times \vec{v}_A) = (10 \text{ kg})(0.25 \text{ m}) (0.25 \text{ m/s}) \hat{j}$

$I_{PA} = 0.3125 \text{ kg m}^2/\text{s}$

$I_{PB} = m_B(\vec{r}_B \times \vec{v}_B) = (5 \text{ kg})(0.5 \text{ m}) (0.5 \text{ m/s}) \hat{j}$

$I_{PB} = 0.625 \text{ kg m}^2/\text{s}$

Linear Momentum

Find magnitude & direction for each mass (A & B).
Remember $\vec{p} = m\vec{v}$

$\vec{p}_A = m_A \vec{v}_A$

$\vec{p}_A = (10 \text{ kg})(0.25 \text{ m/s} \uparrow) = 2.5 \text{ kg m/s} \uparrow$

$\vec{p}_A = (5 \text{ kg})(0.5 \text{ m/s} \downarrow) = 0.5 \text{ kg m/s} \downarrow$

(b)
(b) If pulleys are locked:
\[ \sum M_P = M_{PA} - M_{PB} \]
\[ M_{PA} = \frac{v_A}{v_A} \times F_A \]
\[ M_{PB} = \frac{v_B}{v_B} \times F_B \]

\[ \sum M_P = 0 \]

\[ F \text{ is simply the force exerted on the pulley system by each block. So FBD} \]

\[ m_A g = F_A \]
\[ F_B = m_B g \]

\[ \sum M_P = \left[ \left( \frac{0.25}{2} \right) m \right] (10 \, kg) (9.81 \, m/s^2) - \left[ \left( \frac{0.5}{2} \right) m \right] (5 \, kg) (9.81 \, m/s^2) \]
Known: Jet of water with density \( \rho \) hits hinged flap with mass \( m \). Velocity of water is \( V_{\text{jet}} \). Incoming jet is circular with diameter \( d \).

Find (a) \( \theta \) that stationary flap makes horizontal
(b) horizontal and vertical reaction forces @ \( A \).

Assume \( \theta \) is known from (a).

![Diagram of water jet hitting a hinged flap]

(a) Start w/ Conservation of mass on system above

\[
\frac{d(m_{\text{sys}})}{dt} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}}
\]

Assume steady state:

\[
\dot{m} = \dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \rho A V_{\text{jet}}
\]

\[
\dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \rho \frac{\pi d^2}{4} V_{\text{jet}}
\]

Cons AM about \( A \)

\[
\frac{dL_A}{dt} = \sum F_A + \sum (F \times V)_{\text{in}} - \sum (F \times V)_{\text{out}}
\]

\[
0 = -L_1 \cos(\theta) (mg) + L_2 \sin(\theta) V_{\text{jet}} m - 0 \quad \text{b/c exiting } V_{\text{jet}} \text{ is collinear w/ } A
\]
\[ O = -L_1 \cos(\theta) mg + L_2 \sin(\theta) V_{jet} \rho \frac{\pi d^2}{4} V_{jet} \quad \text{Solve for } \theta \]

\[ \tan \theta = \frac{L_1 mg}{L_2 \sin V_{jet}} = \frac{L_1 mg}{L_2 \rho \frac{\pi d^2}{4} V_{jet}^2} \]

\[ \theta = \tan^{-1} \left( \frac{L_1 mg}{L_2 \rho \frac{\pi d^2}{4} V_{jet}^2} \right) \]

(b) Use Cons Lin Mom

\[ \frac{d T_{sys}}{dt} = \Sigma F + \Sigma \dot{m} \ln V - \Sigma \dot{m}_{out} V \]

\[ x \text{-direction} \]

\[ O = A_x + \dot{m} V_{jet} - \dot{m}_{in} (V_{jet} \cos \theta) \quad \text{Solve for } A_x \]

\[ A_x = \dot{m} (V_{jet} \cos(\theta)) - \dot{m} V_{jet} \]

\[ A_x = \dot{m} V_{jet} (\cos(\theta) - 1) \]

\[ A_x = \rho \frac{\pi d^2}{4} V_{jet}^2 (\cos(\theta) - 1) \]
y-direction

\[ 0 = A_y - mg - \frac{m}{V_{\text{jet}} \sin(\theta)} \]  
\[ A_y = mg - \frac{m}{V_{\text{jet}} \sin(\theta)} \]

\[ A_y = mg - \frac{\pi d^2}{4} V_{\text{jet}}^2 \sin(\theta) \]
Known: Monorail moves on a track

Find: Maximum acceleration for car

Given:

\[ \mu_s = 0.6 \text{ wheel to track} \]

Analysis: Apply LM & AM to car (system defined above)

Car moves b/c of \( F_f \) between wheel + track, ... Assume wheels do not slip

\[ F_{f,\text{max}} = \mu_s A_y \]

Cons Lin Mom

\[ \frac{dP_{sys}}{dt} = \Sigma F_x + \Sigma F_y \quad \text{no mass flow} \]

\[ \Sigma F_x = F_f \]

Remember \( P = mV \) and \( m \) isn’t changing, so

\[ m \frac{dV_{max}}{dt} = F_{f,\text{max}} \]

\[ m \frac{dV_{max}}{dt} = \mu_s A_y \quad (1) \]
\[ \frac{dP_{y, x}}{dt} = Ay + By - mg \]

\[ Ay + By = mg \quad (2) \]

Solve for a_max

\[ \frac{m dV_{\text{max}, x}}{dt} = M_s (mg - By) \quad \text{2 unknowns} \quad (V_{\text{max}}, By) \]

Do Cons. Ang mom about G

\[ \frac{dL_{y, x}}{dt} = \sum_{g} M_g + \sum_{f} (r \times F)_{\text{min}} - \sum_{f} (r \times F)_{\text{out}} \]

\[ \frac{dL_{y, x}}{dt} = Ay(18ft) - By(18ft) - F_f (1ft) \]

\[ 0 = Ay - By - \frac{7}{18} F_f \quad \text{plug in} \quad F_f = M_s Ay \]

\[ 0 = Ay - By - \frac{7}{18} M_s Ay \quad (3) \]

Combine (3) \& (2)

\[ Ay + 0.7 \times 0.5066 \text{ Ay} = mg \quad \Rightarrow \quad Ay = 0.5066 mg \]

\[ By = 0.434 \text{ mg} \]

\[ \frac{dV_{\text{max}, x}}{dt} = \frac{M_s}{m} \text{ Ay} = \frac{0.6}{m} (0.5066 \text{ mg}) \]

\[ = 0.33968 \]
Known: China has now joined the USA & Russia in putting a man in space.

Problem: You must always be P.O.

Find: a) Max height "h" above lunar surface pilot can shut off lunar module if lunar module velocity relative to the moon is:
   i) 0
   ii) 3 m/s up
   iii) 3 m/s down

Using the Work-Energy Principle:

b) Repeat a) using Cons lin Mom

Given:

\[ g_{moon} = \frac{g_{earth}}{6} \]

\[ V_{max} \leq 5 \text{ m/s} \]

Analysis:
a) System: lunar module

Use Work-Energy

\[ W_{mech}(1 \rightarrow 2) = KE_2 - KE_1 + PE_2 - PE_1 \]

\[ KE = \frac{1}{2} m V^2 \]

\[ PE = mgz \]

So,

\[ W_{mech}(1 \rightarrow 2) = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2 + mg_0 \frac{z_2}{h} - mg_0 \frac{h}{0} \]

0 b/c no contact force doing work

Make proper substitutions (shown) and get

\[ 0 = \frac{1}{2} m V_{max}^2 - \frac{1}{2} m V_1^2 - mg_0 h \]

Solve for \( h \)

\[ mg_0 h = \frac{1}{2} m (V_{max}^2 - V_1^2) \]

\[ h = \frac{1}{2} \frac{mg_0}{V_{max}^2 - V_1^2} (V_{max}^2 - V_1^2) = \frac{1}{2} \frac{V_{max}^2 - V_1^2}{g_0} \] (EQN 1)
Solve EQU. 1 for each situation listed in (a).

i) \[ h = \frac{1}{2} \left( \ell^2 - 0 \right) \frac{w^2}{g} = \frac{9.81}{6} \frac{w^2}{g} = 7.15 \text{m} \]

ii) \[ h = \frac{1}{2} \left( 5^2 - 3^2 \right) \frac{w^2}{g} = \frac{9.81}{6} \frac{w^2}{g} = 4.89 \text{m} \]

iii) \[ h = \frac{1}{2} \left( 5^2 - 3^2 \right) \frac{w^2}{g} = \frac{9.81}{6} \frac{w^2}{g} = 4.89 \text{m} \]

(b) System: lunar module
Use Line Mom
Time: Finite

\[
\frac{dP_{\text{sys}}}{dt} = \Sigma F + \Sigma \text{in} - \Sigma \text{out}
\]
Closed system

Remember \( P = mV \) so

\[
\frac{d}{dt} \left( mV \right) = m \frac{dV}{dt} = \Sigma F
\]
\[ \gamma \frac{dV_z}{dt} = -\gamma g_{moon} \quad \text{From Kinematics:} \quad \frac{dV_z}{dt} = V_z \frac{dV_z}{dz} \]

\[ V_z \frac{dV_z}{dz} = -g_{moon} \]

\[ \int_{V_{z_1}}^{V_z} V_z \, dV_z = \int_{z: h}^{2: 0} -g_{moon} \, dz \]

\[ \frac{V_z^2 - V_{z_1}^2}{2} = -g_{moon}(0 - h) \quad \text{Solve for } h \]

\[ h = \frac{\frac{1}{2}(N_{max}^2 - V_z^2)}{g_{moon}} = \text{Same as } \text{EAM, } \text{Thus} \]

\[ \text{i}) \quad 7.65 \text{ m} \]
\[ \text{ii}) \quad 4.89 \text{ m} \]
\[ \text{iii}) \quad 4.89 \text{ m} \]
Known Conditions of a closed system

Find Change in
a) KE of the system in ft·lbf
b) PE ...
c) U ...
4 BTU

Given Closed system undergoes a process

\( m = 5 \text{ lbm} \)

\( Q_{\text{out}} (\text{in}) = 200 \text{ ft·lbf} \)

\( W_{\text{in}} = 0 \)

\( g = 32 \text{ ft/s}^2 \)

\( V_{\text{sys}} = 10 \text{ ft/s} \)

\( V_{\text{sys}} = 50 \text{ ft/s} \)

\( z_2 = z_1 - 150 \text{ ft} \)

Analysis

\( V_1 = 10 \text{ ft/s} \)

\( V_2 = 50 \text{ ft/s} \)

Cons Energy

\( \frac{dE_{\text{sys}}}{dt} = Q_{\text{in}} + W_{\text{in}} + \sum W_{\text{in}}(e_{\text{in}} + P_{\text{in}} + V_{\text{in}}) - \sum W_{\text{out}}(e_{\text{out}} + P_{\text{out}} + V_{\text{out}}) \)

0 Given

Closed System
\( E_{sys2} - E_{sys1} = Q_{in} = - \left( Q_{out1} / 1 \right) \)

(a) Apply (1) to KE

\[
\Delta KE = KE_2 - KE_1 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \frac{1}{2} s \left( 50^2 - 10^2 \right) \frac{\text{lbm-ft}^2}{s^2}
\]

\[
= 6000 \frac{\text{lbm}}{s^2} \cdot \frac{\text{ft}^2}{32.2 \frac{\text{lbm-ft}}{s^2}} = 186.3 \text{ ft-lbf}
\]

(b) Apply (1) to PE

\[
\Delta PE = PE_2 - PE_1 = m g z_2 - m g z_1 = (5)(32)(150)
\]

\[
= 24,000 \frac{\text{lbm}}{s^2} \cdot \frac{\text{ft}^2}{32.2 \frac{\text{lbm-ft}}{s^2}} = -745.3 \text{ ft-lbf}
\]

(c) \( E_{sys2} - E_{sys1} = (U_2 - U_1) + (KE_2 - KE_1) + (PE_2 - PE_1) \)

Assume all other modes of energy unimportant

Now from (1):

\[
U_2 - U_1 + \Delta KE + \Delta PE = - Q_{out1-2}
\]

\[
U_2 - U_1 = - \Delta KE - \Delta PE - Q_{out1-2} = -186.3 - (-745.3) - 200 = 354 \text{ ft-lbf}
\]

Convert 1 ft-lbf to BTU

\[
354 \text{ ft-lbf} \cdot \frac{\text{BTU}}{778 \text{ ft-lbf}} = 0.461 \text{ BTU}
\]
Known: Spring loaded boot-on-a-stick kicks a marble

Find:
(a) an expression for \( V_{boot} \) right before it kicks the marble
(b) assuming the boot & the marble stick together, find an expression for \( V_{marble} \) right after it gets kicked
(c) if the spring was initially compressed \( \frac{d}{3} \) before the device was loaded, would \( V_{boot} \) increase, decrease, or remain the same? Why?

Given:

\[ \begin{align*}
A_x &= 4 \\
A_y &= \text{Spring constant } k \\
\text{compressed distance } d \\
\text{system includes spring system @ 1} \\
\text{system @ 2} \\
g \\
m_b g \\
\text{not moving } D, z_2 = 0 \\
D, no spring @ 2 \\
\text{connection @ A does no work} \\
\end{align*} \]

Analysis:

(a) Work-Energy Principle

\[ W_{12} = KE_2 - KE_1 + PE_2 - PE_1 + E_{k2} - E_{k1} \]

connection @ A does no work

Remember: \( KE = \frac{1}{2} m V^2 \); \( PE = m g z \); \( E_0 = \frac{1}{2} k x^2 \)
\[ 0 = \frac{1}{2} m_b V_{b2}^2 - m_b g L - \frac{1}{2} k d^2 \quad \Rightarrow \text{Solve for } V_{b2} \]

\[ \frac{1}{2} m_b V_{b2}^2 = m_b g L + \frac{1}{2} k d^2 \]

\[ V_{b2}^2 = \frac{m_b g L + \frac{1}{2} k d^2}{\frac{1}{2} m_b} = 2gL + \frac{kd^2}{mb} \]

(a) \[ V_{b2} = \sqrt{2gL + \frac{kd^2}{mb}} \]

(b) Impact, so use lin. mom. finite

\[ V_3 \quad \Theta \quad 0 \quad V_{b2} \]

Cons lin. mom

\[ \frac{dP_{\text{sys}}}{dt} = \Sigma F + \Sigma \text{int} \mathbf{V}_{\text{in}} - \Sigma \text{int} \mathbf{V}_{\text{out}} \]

\[ \frac{dP_{\text{sys}}}{dt} = F \quad \text{separate & integrate} \]

\[ P_{\text{sys}, 3} - P_{\text{sys}, 2} = \overline{F_{\text{avg}}} \Delta t \]
\[ P_{x3} - P_{x2} = \frac{\gamma}{g} x t \]

\[(m_m + m_b) \dot{V}_3 - m_b V_{b2} = 0 \quad \text{Solve for } V_3\]

\[ V_3 (m_m + m_b) = m_b V_{b2} \]

\[ V_3 = \frac{m_b V_{b2}}{m_m + m_b} \]

\[ V_3 = \frac{m_b}{m_m + m_b} \sqrt{2gL + \frac{kd^2}{m_b}} \]

(1) \( V_{b2} \) will decrease. There is less energy w/d/3 compression, so less energy can be converted to KE.
Known: Air trapped above water in a storage tank

Find: (a) Final P of air in tank
     (b) Work & Q for air as it is compressed

Given:
Air is ideal gas!!

$V_{tank} = 0.4 \text{ m}^3$

1. $V_{air} = (0.4 - 0.3) \text{ m}^3 = 0.10 \text{ m}^3$
   $P_1 = 240 \text{ kPa}$
   $T_1 = 20^\circ C = 293 \text{ K}$

2. $V_{air} = (0.4 - 0.35) \text{ m}^3 = 0.05 \text{ m}^3$
   $T_1 = T_2 \Rightarrow$ constant

Analysis:

(a) $m_{air} = \frac{P_1 V_1}{RT_1} = \frac{P_2 V_2}{RT_2}$
   solve for $P_2$

$P_1 V_1 = P_2 V_2$

$P_2 = \frac{P_1 V_1}{V_2}$

$P_2 = \frac{240 \text{ kPa}}{0.2} = 1200 \text{ kPa}$

(b) for Q & W use E.B. for air

$\frac{dE_{sys}}{dt} = Q_{in} + W_{in}$ closed system

$\Delta U = Q + W$ finite time

$\Delta U \rightarrow 0$ & KE = ΔPE = 0
Now \( \Delta U = m \Delta u = m c_v (T_2 - T_1) = 0 \) \((b/c \ T_2 = T_1)\)

So \( Q_{in} = -W_{in} \)

\[ W_{in} = -\int_1^2 P dV \quad \text{and} \quad P = \frac{mRT}{V} \]

\[ W_{in} = -\int_1^2 \frac{mRT}{V} dV \]

\[ W_{in} = -mRT \int_1^2 \frac{1}{V} dV \]

\[ W_{in} = -mRT \left( \ln V_2 - \ln V_1 \right) \]

\[ W_{in} = -mRT \ln \left( \frac{V_2}{V_1} \right) \]

\[ W_{in} = -P_1 V_1 \ln \left( \frac{V_2}{V_1} \right) \quad \text{Plug in & solve} \]

\[ W_{in} = 16.64 \text{ KJ} \]

\[ Q_{in} = -16.64 \text{ KJ} \]
Known
Centrifugal pump driven by a motor

Find
(a) Heat transfer rate for the pump, kW
(b) Torque for the shaft, N·m
(c) Electric current to motor, amps

Given

<table>
<thead>
<tr>
<th></th>
<th>①</th>
<th>②</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>20°C</td>
<td>20°C</td>
</tr>
<tr>
<td>P</td>
<td>100 kPa</td>
<td>500 kPa</td>
</tr>
<tr>
<td>A</td>
<td>180 cm²</td>
<td>125 cm²</td>
</tr>
<tr>
<td>V</td>
<td>5 m/s</td>
<td>?</td>
</tr>
</tbody>
</table>

N_{shaft} = 1750 \text{ rpm}

W_{shaft} = 40 \text{ kW} \quad \text{W_{elec} = 42 \text{ kW}}

440 Volt ac motor \quad \text{Power Factor} = 1

Analysis

Strategy — Conservation of Mass & Energy

\[ \frac{dE_{sys}}{dt} = Q_{in} + W + m_1 \left( h_1 + \frac{V_1^2}{2} + gz_1 \right) - m_2 \left( h_2 + \frac{V_2^2}{2} + gz_2 \right) \]

\[ Z_1 = Z_2 \]

\[ \therefore Q_{out} = W_{in} + m_1 \left( h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} \right) \]

Less mass balance \[ m_1 = m_2 \]
\[ \dot{m} = \rho A V_1 = \left(997 \text{ kg/m}^3\right) \left(180 \times 10^{-4} \text{ m}^2\right) (5 \text{ m/s}) \]

\[ = 89.73 \text{ kg/s} \]

\[ \dot{m}_1 = \dot{m}_2 \quad \Rightarrow \quad \dot{V}_1 = \dot{V}_2 \quad \Rightarrow \quad A_1 V_1 = A_2 V_2 \quad \rho \text{ constant} \]

\[ V_2 = \frac{A_1}{A_2} V_1 \]

\[ V_2 = 7.20 \text{ m/s} \]

\[ A h = c \Delta T + V \Delta P \]

\[ T_1 = T_2 \]

\[ \therefore \dot{Q}_{\text{out}} = \dot{W}_{\text{in}} + \dot{m} \left( V (P_1 - P_2) + \frac{V_1^2 - V_2^2}{2} \right) \quad \text{plugin #}'s \]

\[ \dot{Q}_{\text{out}} = 40 \text{ kW} + 89.73 \text{ kg/s} \left[ \frac{100-500 \text{ kPa}}{997 \text{ kg/m}^3} + \frac{(5^2-7.2^2)}{2} \frac{\text{m}^2}{\text{s}^2} \right] \]

\[ \dot{Q}_{\text{out}} = 2.80 \text{ kW} \]

Final Torque

\( (b) \quad \tau = \frac{W_{\text{shaft}}}{\omega} \quad \omega = \frac{1750 \text{ rev}}{\text{min}} = \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 183.3 \text{ rad/s} \)

\[ \tau = \frac{40,000 \text{ W}}{183.3 \text{ rad/s}} = 218.3 \text{ N.m} \]
(c) Finding Current

\[ W_{elec} = i \Delta V \]

\[ i = \frac{W_{elec}}{\Delta V} \]

\[ i = \frac{4\,200\,000 \text{ W}}{1410 \text{ Vols}} = 95.46 \text{ amps} \]
Known: Gearbox operates at steady state

Find: (a) $Q$ (in or out) in B/h
(b) $Q_{kin}$ for gearbox & shafts
(c) " " air layer
(d) " " air layer & gearbox & shafts as a system
(e) comment on (b), (c), & (d)

Given:

![Diagram]

- Air layer
- Gearbox
- 2 HP
- 1.9 HP
- $T_s = 105^\circ C$
- $T_{Room} = 70^\circ F$

Analysis:

(a) Gearbox is system; use Conservation of Energy

$$\frac{dE_{sys}}{dt} = Q_{in} + W_{in} + \int_{0}^{\infty} \ldots \quad \text{no mass flow} \quad \text{and no mass flow}$$

$$0 = Q_{in} + W_{in}$$

$$Q_{in} = - W_{in}$$

$$Q_{in} = -(2 HP - 1.9 HP)$$

$$Q_{in} = -0.1 HP \quad \text{so must be } Q_{out} \text{ (b/c negative)}$$
(a) $Q_{out} = 0.1 \text{ HP}$ convert to $\frac{B}{h}$

$$0.1 \text{ HP} \cdot 2546 \frac{B}{\text{HP}} = 254.6 \frac{B}{h}$$

(b) Entropy - System is geared shafts

$$\frac{dS_{sys}}{dt} = \sum \frac{Q_i}{T_j} + \sum \frac{Q_i}{T_j} - \sum \frac{Q_f}{T_f} + \dot{S}_{gen}$$

0 Steady State

$$0 = \frac{Q_i}{T_j} + \dot{S}_{gen} \quad \text{Solve for } \dot{S}_{gen}$$

$$\dot{S}_{gen} = -\frac{Q_i}{T_j}$$

$$\dot{S}_{gen} = -254.6 \frac{B}{h} \frac{B}{105 + 460} = 0.451 \frac{B}{h \circ K}$$

(c) System is air layer

Conservation of Energy

$$\frac{dE_{sys}}{dt} = Q_{in} + W_{in} + \sum \frac{Q_i}{T_j} - \sum \frac{Q_f}{T_f}$$

0 (SS)

$$0 = Q_A - Q_B$$

$$Q_B = Q_A = 254.6 \frac{B}{h} \quad \text{(makes sense)}$$
Entropy Accounting

\[ \frac{d S_{sys}}{dt} = \sum \frac{Q_i}{T_j} + \sum S_i - \sum S_i + S_{gen} \]

\[ 0 = \sum \frac{Q_i}{T_j} + S_{gen} \]

\[ S_{gen} = -\frac{Q_i}{T_j} \]

\[ S_{gen} = -\left[ \frac{Q_A}{T_A} - \frac{Q_B}{T_B} \right] = \left[ \frac{254.6}{105 + 460} - \frac{254.6}{10 + 460} \right] \]

\[ S_{gen} = 0.0297 \frac{B}{h-oR} \]

(d) System is now gearbox w/air layer

Entropy Accounting

\[ \frac{d S_{sys}}{dt} = \sum \frac{Q_i}{T_j} + \sum S_i - \sum S_i + S_{gen} \]

\[ 0 = \sum \frac{Q_i}{T_j} + S_{gen} \]

\[ S_{gen} = -\frac{Q_i}{T_j} = -\frac{254.6 \frac{B}{h}}{105 + 460} \]

\[ S_{gen} = 0.48 \frac{B}{h-oR} \]

(c) From (b) + (d),

\[ S_{gen, gearbox + air layer} = S_{gen, gearbox} + S_{gen, air layer} \]
Known: Jet engine nozzle, adiabatic

Find:
(a) $T_{\text{out}}$, $V_{\text{out}}$ if internally reversible
(b) $S_{\text{gen}}$, $V_{\text{out}}$ if $T_{\text{out}} = 527^\circ C$
(c) compare $T_2$, $V_2$, & $S_{\text{gen}}$

Given:
\[
\begin{align*}
\text{in} & \quad \rightarrow \\
180 \, \text{kPa} & \quad 70 \, \text{kPa} \\
707^\circ C & \quad \text{in} = 3.0 \, \text{kg/s} \\
70 \, \text{m/s} & \quad C_p = 1.00 \, \text{kJ/kgK}
\end{align*}
\]

Assume:
Steady state; ideal gas; negligible change in potential energy

Strategy:
Cons Energy, 2nd law, open system
\[
\frac{dE_{\text{sys}}}{dt} = \dot{Q}_{\text{in}} + \dot{W}_{\text{do}} + \dot{m}_{\text{in}} \left( h + \frac{V^2}{2} + g z \right)_{\text{in}} - \dot{m}_{\text{out}} \left( h + \frac{V^2}{2} + g z \right)_{\text{out}}
\]

\[
\dot{m}_{\text{in}} + \frac{V_{\text{in}}}{2} = \dot{m}_{\text{out}} + \frac{V_{\text{out}}}{2}
\]  \hspace{1cm} (1)
\[ \frac{dS_{sys}}{dt} = \sum_{j} \frac{Q_{j}}{T_{j}} + \dot{m}(S_{in} - S_{out}) + \dot{S}_{gen} \]

\[ 0 = \dot{m}(S_{in} - S_{out}) + \dot{S}_{gen} \]

\[ \dot{S}_{gen} = \dot{m}_{out} - \dot{m}_{in} \]

(a) When system is reversible \( \dot{S}_{gen} = 0 \) so

\[ 0 = \dot{m}_{in}S_{out} - \dot{m}_{in}S_{in} \]

\[ \dot{m}_{in} = \dot{m}_{out} \]

\[ S_{in} = S_{out} \quad \text{or} \quad \frac{S_{2} - S_{1}}{S_{2} - S_{1}} = 0 \]

\[ S_{2} - S_{1} = C_{p} \ln \left( \frac{T_{2}}{T_{1}} \right) - R \ln \left( \frac{P_{2}}{P_{1}} \right) = 0 \]

\[ C_{p} \ln \left( \frac{T_{2}}{T_{1}} \right) = R \ln \frac{P_{2}}{P_{1}} \]

\[ \ln \frac{T_{2}}{T_{1}} = \frac{R}{C_{p}} \ln \frac{P_{2}}{P_{1}} \]

\[ \left( \frac{T_{2}}{T_{1}} \right) = \left( \frac{P_{2}}{P_{1}} \right)^{\frac{R}{C_{p}}} = \left( \frac{70}{180} \right)^{0.287/1} = 0.76257 \]

\[ T_{2} = (707 + 273) \times 0.76257 = 743.3 \text{ K} = 474 \text{ °C} \]

\[ T_{2} = 474 \text{ °C} \]
Energy: \( h_{in} - h_{out} = \frac{V_{out}^2 - V_{in}^2}{2} \)  

Solve for \( V_{out} \)

\[ V_{out}^2 = V_{in}^2 + Z (h_{in} - h_{out}) \]

\[ V_{out}^2 = V_{in}^2 + Z (C_p (T_{in} - T_{out})) \]

\[ V_{out} = \sqrt{(10 \text{ m/s})^2 + Z \left[ (1.00 \text{ kJ/kg}) (707 - 474) \right]} \times 1000 \]

\[ V_{out} = 6.86 \text{ m/s} * \text{if the system is reversible} * \]

(b) \( \dot{S}_{gen} = \dot{m} (s_{out} - s_{in}) \)

\[ s_{out} - s_{in} = C_p \ln\left( \frac{T_2}{T_1} \right) - R \ln\left( \frac{P_2}{P_1} \right) \]

\[ = 1.0 \ln\left( \frac{527 + 273}{707 + 273} \right) - 0.287 \ln\left( \frac{70}{180} \right) \]

\[ \dot{m} = 3 \text{ kg/s} \]

\[ \dot{S}_{gen} = 0.204 \text{ kW/K} \]

\[ V_{out} = \sqrt{70^2 + 2 \left[ (1.00 \text{ kJ/kg}) (707 - 527) \right]} \times 1000 \]

\[ V_{out} = 6.04 \text{ m/s} \]
Reversible

$474^\circ C$  $T_2$  $527^\circ C$

$686 \text{ m/s}$  $V_2$  $604 \text{ m/s}$

The irreversible case has lower $V_2$ & higher $T_2$.

The purpose of a nozzle is to convert internal energy ($u$) and flow work ($pV$) into kinetic energy ($\frac{V^2}{2}$). The reversible nozzle does a better job of this than the irreversible one.
Known: Electric water heater

Find:
(a) $W_{in, elec}$
(b) $Q_{in}$ water only
(c) $Q_{in}$ water & resistor
(d) why do results differ?

Given

\begin{align*}
\text{Water heater} \\
W_{in, elec} & \frac{3}{4}
\end{align*}

\begin{align*}
T_1 &= 18^\circ C \\
T_{resistor} &= 97^\circ C \\
\rho &= 1000 \text{ kg/m}^3 \\
C &= 4.18 \text{ kJ/kg.K} \\
V &= 100L = 0.1 \text{ m}^3 \\
T_2 &= 60^\circ C
\end{align*}

Assume incompressible; no $Q_{out}$ of heater; no storage in resistor or water

Strategy: Cons Energy & 2nd law - finite time; closed system

\begin{align*}
E_{sys, 2} - E_{sys, 1} &= Q_{1-2} + W_{1-2} \\
S_{sys, 2} - S_{sys, 1} &= \frac{Q_{in}}{T_{in}} + S_{qin}
\end{align*}

Solve:
(a) $E_{sys, 2} - E_{sys, 1} = U_2 - U_1 = W_{1-2}$

system: water & resistor
$$U_2 - U_1 = m (U_2 - U_1) = m (c (T_2 - T_1)) = \rho \cdot c (T_2 - T_1)$$

$$U_2 - U_1 = 1000 \cdot (0.1) \cdot (418) \cdot (60 - 18)$$

$$U_2 - U_1 = W_{1-2}, \text{elec} = 17560 \, \text{kJ}$$

(b) System: water only (not including the resistor)

$$Q_{in} = W_{1-2}, \text{elec} = 17560 \, \text{kJ}$$

from cons energy applied to resistor only

$$S_{gen} = S_1 - S_2 = \frac{Q_{in}}{T_{surf,in}} = m (S_2 - S_1) - \frac{Q_{in}}{T_{surf,in}}$$

$$S_2 - S_1 = c_p \cdot \ln \left( \frac{T_2}{T_1} \right) = 41.18 \cdot \ln \left( \frac{273 + 60}{273 + 18} \right)$$

$$T_{surf,in} = 97^\circ \text{C} = 370 \, \text{K}$$

$$S_{gen} = \rho \cdot c_p \cdot \ln \left( \frac{T_2}{T_1} \right) - \frac{Q_{in}}{T_{surf,in}} = 8.91 \, \text{kJ/K}$$

(c) System: water + resistor \quad \therefore \quad Q = 0

$$S_{gen} = m (S_2 - S_1) = \rho \cdot c_p \cdot \ln \left( \frac{T_2}{T_1} \right)$$

$$S_{gen} \approx 5 \, \text{kJ/K}$$

(d) (c) includes resistor, (b) does not

There is heat transfer from the hot resistor to the cool water (which accounts for the $S_{gen}$ discrepancy between the two).