Math 221 Sections 3 and 4 Least Squares Homework
Due date: Friday, January 30

Instructions: You may use Maple except for the command

>leastsqrs(A,b);

Please turn in your Maple worksheet.

1. (From Elementary Linear Algebra with Applications, by Nicholson) A naturalist measured the heights $y_i$ (in meters) of several spruce trees with trunk diameters $x_i$ (in centimeters). The data are as given in the table. Find the least-squares approximating line for these data, and use it to estimate the height of a spruce tree with a trunk diameter of 10 cm (To estimate this height, simply substitute $x = 10$ into your least-squares line). Note: $x$ and $y$ do not have to have the same units to do this problem, e.g. there is no need to convert all measurements to meters or all measurements to centimeters; just use the numbers as they are.

\[
\begin{array}{cccccc}
  x_i & 5 & 7 & 8 & 12 & 13 & 16 \\
  y_i & 2 & 3.3 & 4 & 7.3 & 7.9 & 10.1 \\
\end{array}
\]

Answer: $y = 0.75x - 1.87$, 5.6 meters.

2. On an planet that somewhat resembles earth, an object is thrown vertically (either up or down) from a building. Its height $h$ is given by

\[
h = \frac{1}{2}gt^2 + v_0t + h_0
\]

where $g$ is the acceleration due to gravity, $v_0$ is the initial velocity, and $h_0$ is the initial height. The following measurements of the object’s height at various times were made:

\[
\begin{array}{cccc}
  \text{time (s)} & 1 & 2 & 3 & 4 \\
  \text{height (m)} & 107 & 110.2 & 99.9 & 83.7 \\
\end{array}
\]

Use the least-squares procedure to estimate $g$, $v_0$, and $h_0$. Answer: $g = -9.7$ m/s$^2$, $v_0 = 16.2$ m/s, $h_0 = 96$ m.