Section 7.3 Estimation and tests for Comparing Two Population Variances

- Suppose that we have two independent random samples of size $n_1$ and $n_2$ drawn from two normal populations.
- Let $s_1^2$ and $s_2^2$ be the sample variances, and let $\sigma_1^2$ and $\sigma_2^2$ be the population variances.
- The statistic
  \[ F = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \]
  has an $F$-distribution with
  \[ df_1 = \text{degrees of freedom in the numerator} = n_1 - 1 \]
  \[ df_2 = \text{degrees of freedom in the denominator} = n_2 - 1 \]
- An $F$-distribution curve looks somewhat like the graph in figure 1:
Figure 2:

Some notation:
$F_{\alpha, df_1, df_2}$ = the location on the horizontal axis that cuts off a right-tail area of $\alpha$ underneath the $F$-distribution curve with degrees of freedom $df_1, df_2$. See figure 2: The shaded area is $\alpha$, and the point on the horizontal axis where the shaded area begins is $F_{\alpha, df_1, df_2}$.

Hypothesis testing
Suppose we want to test $H_0 : \sigma_1^2 = \sigma_2^2$ against one of the following alternatives:

1. $H_\alpha : \sigma_1^2 \neq \sigma_2^2$
2. $H_\alpha : \sigma_1^2 > \sigma_2^2$

Test statistic:
- Alternative hypothesis 1. $F =$ larger $s_1^2$/smaller $s_2^2$. (Different from what the textbook says).
- Alternative hypothesis 2. $F = s_1^2/s_2^2$

When to reject $H_0$:
- Alternative hypothesis 1: Reject $H_0$ if $F \geq F_{\alpha/2, df_1, df_2}$, where $df_1 =$ degrees of freedom in the numerator of (larger $s^2$/smaller $s^2$), and $df_2 =$ degrees of freedom in the denominator of (larger $s^2$/smaller $s^2$). This seems different from the textbook’s method but it gives the same results.
- Alternative hypothesis 2: Reject $H_0$ if $F \geq F_{\alpha, df_1, df_2}$, where $df_1 =$ degrees of freedom in the numerator of $s_1^2/s_2^2$, and $df_2 =$ degrees of freedom in the denominator of $s_1^2/s_2^2$. 

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Example 1: In the Minitab worksheet "May 6 lecture", the IQ scores for 31 seventh-grade girls and 41 seventh-grade boys are given. Test the research hypothesis that IQ scores for seventh grade girls have a different spread than those for seventh-grade boys. Use $\alpha = 0.05$. 
Example 2: In semiconductor manufacturing, wet chemical etching is often used to remove silicon from the backs of wafers prior to metalization. The etch rate is an important characteristic in this process and known to follow a normal distribution. Two different etching solutions have been compared, using two random samples of 10 wafers for each solution. The observed etch rates are given in the Minitab worksheet "May 6 lecture". Test the research hypothesis that the etch rates of solution 1 have greater variance than the etch rates of solution 2. Use $\alpha = 0.05$. 