More on independence

Example 1a. Let $A$ and $B$ be events in the same sample space. Prove that $P(A \cap B) = P(A) - P(A \cap B)$.

1b. Let $A$ and $B$ be independent events. Show that the pairs $(A, B), (A, B)$, and $(A, B)$ are also independent.

Independence of more than two events

Let $A_1, A_2, \ldots, A_n$ be events. We say that $A_1, A_2, \ldots, A_n$ are independent if for any selection of events $A_m, A_m, \ldots, A_m$ of events from the set $A_1, A_2, \ldots, A_n$,

$$P(A_m \cap A_m \cap \ldots \cap A_m) = P(A_m)P(A_m) \cdots P(A_m).$$

Example 2: 60% of all vehicles examined at a certain emissions inspection station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, calculate the following:

(a) $P$(The next three vehicles pass inspection)
Example 3: James Bond trainee 007-01 has probability 0.7 of hitting his intended target. James bond trainee 007-02 has probability 0.8 of hitting this same target. If each fire one shot at this target, what is the probability that at least one of them will hit it?

Section 4.6 Random Variables

(See handout on random variables in public directory).

Definition: A random variable $X$ is a rule that assigns a real number to each outcome in the sample space of an experiment. A random variable is usually denoted by a capital letter.

• In this class, random variable will always represent a quantitative variable.

Example 4: Examples of random variables.
(a) Suppose that an experiment consists of tossing a fair coin twice. Let $X$ denote the number of heads that appears.
(b) Let $T$ be the temperature at the airport at 7:30 in the morning.
(c) Let $N$ denote the number of calls received by an operator between 1:00 PM and 2:00 PM today.

Definition: Countable sets
A set $S$ is countable if its members can be written as a (finite or infinite list).

Example 5: Some countable sets.
$S = \{1, 2, 3, \ldots\}$
$S =$ possible outcomes when rolling two dice $= \{(1, 1), (1, 2), (1, 3), \ldots, (6, 5), (6, 6)\}$. 
Definition: Discrete random variable

A random variable $X$ is discrete if all of its possible values comprise a countable set.

Example 6: Some discrete random variables.

- Let $X$ denote the number of siblings that a randomly selected student from the class has. Let’s say $X$ can take the values $\{0, 1, 2, \ldots, 20\}$
- Let $X$ denote the number of times you have to toss a coin before you get a head. Then $X$ can take the values $1, 2, 3, \ldots$. 