Statistical testing: The basic idea

Recap of statistical testing (or hypothesis testing) for population mean $\mu$:

1. Begin by stating the null hypothesis $H_0$ and the alternative hypothesis $H_a$.
   a. We will say that $H_0$ always has the form:
      $$H_0 : \mu = \mu_0$$
   for some constant $\mu_0$, although the textbook uses a slightly different form for $H_0$. (More on this later).
   b. We will say that $H_a$ always has one of the following forms:
      $$H_a : \mu < \mu_0, \ H_a : \mu > \mu_0, \ \text{or} \ H_a : \mu \neq \mu_0$$

2. Make sure that a significance level $\alpha$ is given.
3. Select a random sample of size $n$ from the population and let $\overline{x}$ be the sample mean.
4. Compute the test statistic $z_0 = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}$.
5. Let $p$ be the probability that we would observe a value of $z$ as extreme or more extreme than $z_0$. (We will define "more extreme" shortly).
6. If $p \leq \alpha$, reject $H_0$ in favor of $H_a$. Otherwise, our evidence (namely the observation $z_0$) is not strong enough to reject $H_0$.

Explanation of step 5: The meaning of “more extreme” depends on $H_a$.

- If $H_a$ has the form $\mu > \mu_0$, then $z$ is “more extreme” than $z_0$ if $z \geq z_0$.
- If $H_a$ has the form $\mu < \mu_0$, then $z$ is “more extreme” than $z_0$ if $z \leq z_0$.
If $H_a$ has the form $\mu \neq \mu_0$, then $z$ is “more extreme” than $z_0$ if $z$ is farther from $\mu_0$ than $z_0$.

**Example 1:** (From Moore, The Basic Practice of Statistics). The National Center for Health Statistics reports that the mean systolic blood pressure for males 35 to 44 years of age is 128 and the standard deviation in this population is 15. The medical director of a large company looks at the medical records of 72 executives in this age group and finds that the mean systolic blood pressure in this sample is $\bar{x} = 126.07$. Is this sufficient evidence that the company’s executives have a different mean blood pressure from the general population? Assume that executives have the same $\sigma = 15$ as the general population. Use $\alpha = 0.05$. 
Example 2: Weekly sales of ground coffee at a supermarket have, in the recent past, been $N(354, 33)$. The store reduces the price by 5%. Sales in the next three weeks are 405, 378, and 411 units. Is this good evidence that sales are now higher, if $\alpha = 0.05$? What if $\alpha = 0.01$?
Example 3: In the Minitab worksheet “April_19_lecture.mws”, the measurements of a critical dimension on a sample of automobile engine crankshafts is given in the column “Crankshafts”. The manufacturing process is known to vary normally with $\sigma = 0.060$. The process mean is supposed to be 224 mm. Do these data give evidence that the process mean is not equal to the target value 224 mm? Use $\alpha = 0.01$. 
Rejection regions and acceptance regions.

Suppose that we wish to test $H_0 : \mu = \mu_0$ vs. $H_a : \mu > \mu_0$ with significance level $\alpha$. How big does the test statistic $z$ have to be to reject $H_0$?

The interval $[z_\alpha, \infty)$ is called the rejection region for this test. The interval is called the acceptance region.

Suppose that we wish to test $H_0 : \mu = \mu_0$ vs. $H_a : \mu < \mu_0$ with significance level $\alpha$. How big does the test statistic $z$ have to be to reject $H_0$?

The interval is the rejection region for this test. The interval is the acceptance region.

Suppose that we wish to test $H_0 : \mu = \mu_0$ vs. $H_a : \mu \neq \mu_0$ with significance level $\alpha$. How far from $\mu_0$ does the test statistic $z$ have to be to reject $H_0$?

What are the acceptance and rejection regions?
Example 4: To determine whether the mean nicotine content of a brand of cigarettes is greater than the advertised value of 1.4 milligrams, a health advocacy group tests

\[ H_0 : \mu = 1.4 \]
\[ H_a : \mu > 1.4 \]

Suppose that \( \sigma = 0.4 \) and that we are going to use a random sample of 10 cigarettes to conduct our hypothesis test.

(a) What are the acceptance and rejection regions for \( \alpha = 0.01 \) and \( \alpha = 0.05 \), in terms of \( \bar{x} \)?
(b) Suppose that the calculated value of the test statistic is $x = 1.71$. Without calculating any $P$–values, what do you conclude if $\alpha = 0.01$ and $\alpha = 0.05$?