1. A cola company wants to determine whether its cola loses sweetness after being stored for a month at a high temperature. The cola maker determines that the sweetness loss is too large to accept if the mean sweetness loss for the population of all cans is $\mu = 1$. So the cola maker decides to conduct the hypothesis test

$$H_0 : \mu = 0$$
$$H_a : \mu > 0$$

using a random sample of 10 cans, with $\alpha = 0.05$. Assume that $\sigma = 1$.

In this problem we calculate $\beta$, the probability of a Type II error, given that the true value of $\mu$ is 1.1. Remember that $\beta$ is the probability that we incorrectly fail to reject $H_0$ when the true value of $\mu$ is 1.1. Follow these steps:

**Step 1:** Write the rule for accepting $H_0$ in terms of $\bar{x}$ (the mean sweetness loss of the sample). We know that the test fails to reject $H_0$ when

$$z = \frac{\bar{x} - 0}{1/\sqrt{10}} < 1.645.$$ 

Restate this in terms of $\bar{x}$. This gives the acceptance (of $H_0$) region.

**Step 2:** Find the probability that $\bar{x}$ falls into the acceptance region if $\mu = 1.1$. This probability is $\beta$. Include a rough sketch that illustrates this probability.

2. Bottles of cola are supposed to contain 300 ml of cola. The distribution of the contents is normal with $\sigma = 3$ ml.

Suppose that we conduct a test of

$$H_0 : \mu = 300$$
$$H_a : \mu < 300.$$ 

The sample size is 6, and $\alpha = 0.05$.

(a) Find the acceptance region of the test in terms of $\bar{x}$.

(b) Find $\beta$ if the true population mean is $\mu = 299$. Include a rough sketch illustrating $\beta$.

3. A lab is asked to test the claim that the concentration of the active ingredient in a specimen is 0.86%. The true concentration is the mean $\mu$ of the population of all analyses of the specimen. The lab plans to make 3 repeated analyses of the specimen. The standard deviation of the analysis process is $\sigma = 0.0068$. The lab will test

$$H_0 : \mu = 0.86$$
$$H_a : \mu \neq 0.86$$

with $\alpha = 0.01$. If the true value of $\mu$ is 0.845, what is $\beta$? Include a sketch of the region whose area is $\beta$. 

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