Math 112  Section 10  Lecture notes

Section 6.7 Fluid Pressure and Force (We’ll be skipping section 6.6).

Definition of Fluid Pressure:
The pressure on an object at depth $h$ (where $h \geq 0$) in a liquid is

\[ \text{Pressure} = P = wh \]

where $w$ is the weight-density of the liquid per unit of volume. Pressure is given in units of force per unit area, e.g. lbs/ft$^2$.

In this definition, it is assumed that the depth of the object is constant, i.e. each part of the object is at the same depth.

Fluid force on a submerged horizontal surface:

\[ \text{Fluid force} = F = PA = (\text{pressure})(\text{area}). \]

This formula also works for a submerged vertical surface, as long as its height is negligible so that its depth is practically constant.
1. One of the vertical sides of a water tank is a rectangle with height 4 feet and width 5 feet. Suppose that this side is partitioned into extremely short rectangles with height $dy$.

a. What is the fluid force on a rectangle with depth $-y$ as a function of $y$? The weight density of water is 62.4 lbs/ft$^3$.
   \[ \text{Answer: } (62.4)(-y)5dy = -312y \, dy \, \text{lbs.} \]

b. Find the total fluid force on the side of the tank by "adding up" the force from part a.
   \[ \text{Answer: } \int_{-4}^{0} -312y \, dy = 2496 \, \text{lbs.} \]
2. One of the vertical sides of a water tank is an isosceles triangle with height 6 and width 4. Suppose this side is partitioned into rectangles with negligible height $dy$.

a. What is the area of the rectangle at depth $6 - y$, as a function of $y$?
   \[
   \text{Answer: } \frac{2}{3}y \, dy.
   \]

b. What is the fluid force on the rectangle at depth $4 - y$?
   \[
   \text{Answer: } (62.4)(6 - y)\frac{2}{3}y \, dy
   \]

c. What is the fluid force on the side of the tank?
   \[
   \text{Answer: } \int_{0}^{6} (62.4)(6 - y)\frac{2}{3}y \, dy = 1492.6 \text{ lbs.}
   \]
3. A water tank has a vertical side in the shape of the lower half of a circle with radius 3. This side is partitioned into very short rectangles with height $dy$.

(a) What is the area of the rectangle at depth $-y$ as a function of $y$?
Answer: $2\sqrt{9 - y^2} dy$

(b) What is the fluid force on the rectangle at height $-y$?
Answer: $-y(62.4)2\sqrt{9 - y^2} dy$

(c) What is the fluid force on the side of the tank?
Answer: $\int_{-3}^{0} -y(62.4)2\sqrt{9 - y^2} dy = 1123.2$ lbs.
4. A water tank has a vertical side in the shape of a semiellipse with equation

\[ y = -\frac{1}{2}\sqrt{36 - 9x^2}. \]

This side of the tank is partitioned into very short rectangles with height \( dy \).

(a) What is the area of the rectangle at depth \(-y\) as a function of \( y\)?

\[ \text{Answer: } \frac{4\sqrt{9 - y^2}}{3} dy. \]

(b) What is the fluid force on the rectangle at depth \(-y\)?

\[ \text{Answer: } -\frac{4y\sqrt{9 - y^2}}{3} \] (62.4)

(c) What is the fluid force on the side of the tank?

\[ \text{Answer: } \int_{-3}^{0} -\frac{4y\sqrt{9 - y^2}}{3} dy = 748.8 \] (62.4)