Definition of Fluid Pressure:
The pressure on an object at depth $h$ (where $h \geq 0$) in a liquid is

$$\text{Pressure} = P = \rho h$$

where $\rho$ is the weight-density of the liquid per unit of volume. Pressure is given in units of force per unit area, e.g. lbs/ft$^2$.

In this definition, it is assumed that the depth of the object is constant, i.e. each part of the object is at the same depth.

Fluid force on a submerged horizontal surface:

$$\text{Fluid force} = F = PA = (\text{pressure})(\text{area})$$

This formula also works for a submerged \textit{vertical} surface, as long as its height is negligible so that its depth is practically constant.
1. One of the vertical sides of a water tank is a rectangle with height 4 feet and width 5 feet. Suppose that this side is partitioned into extremely short rectangles with height $dy$.

a. What is the fluid force on a rectangle with depth $-y$ as a function of $y$? The weight density of water is 62.4 lbs/ft$^3$.

   Answer: $(62.4)(-y)(5)dy = -312y dy$ lbs.

b. Find the total fluid force on the side of the tank by "adding up" the force from part a.

   Answer: $\int_{-4}^{0} -312y\,dy = 2496$ lbs.
2. One of the vertical sides of a water tank is an isosceles triangle with height 6 and width 4. Suppose this side is partitioned into rectangles with negligible height $dy$.

**a.** What is the area of the rectangle at depth $6-y$, as a function of $y$?

*Answer:* $\frac{2}{3}y dy$.

**b.** What is the fluid force on the rectangle at depth $4-y$?

*Answer:* $(62.4)(6-y)\frac{2}{3}y dy$

**c.** What is the fluid force on the side of the tank?

*Answer:*

$$\int_0^6 (62.4)(6-y)\frac{2}{3}y dy = 1492.6 \text{ lbs.}$$
3. A water tank has a vertical side in the shape of the lower half of a circle with radius 3. This side is partitioned into very short rectangles with height $dy$.

(a) What is the area of the rectangle at depth $-y$ as a function of $y$?

Answer: $2\sqrt{9 - y^2} dy$

(b) What is the fluid force on the rectangle at height $-y$?

Answer: $-y(62.4)2\sqrt{9 - y^2} dy$.

(c) What is the fluid force on the side of the tank?

Answer: $\int_{-3}^{0} -y(62.4)2\sqrt{9 - y^2} dy = 1123.2$ lbs.
4. A water tank has a vertical side in the shape of a semiellipse with equation

\[ y = -\frac{1}{2}\sqrt{36 - 9x^2}. \]

This side of the tank is partitioned into very short rectangles with height \( dy \).

(a) What is the area of the rectangle at depth \( -y \) as a function of \( y \)?

Answer: \( \frac{4\sqrt{9 - y^2}}{3} \) \( dy \).

(b) What is the fluid force on the rectangle at depth \( -y \)?

Answer: \( -\frac{4y\sqrt{9 - y^2}}{3} \) (62.4)

(c) What is the fluid force on the side of the tank?

Answer: \( \int_{-3}^{0} -\frac{4y\sqrt{9 - y^2}}{3} \) (62.4) \( dy = 748.8 \)