Applied series problems

1. Suppose that you accept a job that pays $40,000 for the first year. Suppose that during the next 39 years you receive a 4% raise each year. What is your total compensation over the 40-year period? Answer: $3,080,020.63.
2. In a certain experiment, a coin is tossed until heads appears.
(a) Let $P_n$ be the probability that the experiment stops after $n$ tosses. Find a formula for $P_n$.

(b) What is the probability that the experiment stops after an even number of tosses? Answer: 1/3.
3a. Let $|r| < 1$. Prove that

$$
\sum_{n=1}^{\infty} nr^{n-1} = \frac{1}{(1-r)^2}.
$$

*Hint:* What is $\frac{d}{dr}(1 + r + r^2 + \ldots)$?

b. Refer to the experiment in problem 2. The average number of tosses until a head appears is

$$
\pi = \sum_{n=1}^{\infty} nP_n,
$$

where $P_n$ is the probability that the first head appears on the $n$th toss. Evaluate $\pi$. *Answer: $\pi = 2$. 