Supplement for Differential Equations

MA112

February 17, 2004

This is a set of typical differential equations problems that all students in MA112 should be able to do by the end of the quarter. The students are expected to set up an appropriate differential equation and solve using separation of variables.

1 Exponential Growth and Decay Problems

1. A certain type of bacteria is known to grow at a rate proportional to the amount that is present. If you start with 2 grams of the bacteria and two hours later there are 2.1 grams, how much will you have after 5 hours?

2. Suppose that 15 grams of Einsteinium were released (the half life of Einsteinium is 270 days). How long will it take for 15 grams to decay to 10 grams?

3. A certain type of bacteria is known to grow at a rate proportional to the square root of the amount that is present. If you start with 3 grams of the bacteria and three hours later there are 3.5 grams, then how much will you have after 7 hours?

4. Suppose that 10 grams of radium were released (the half-life of Radium is 1620 years). How long will it take for the 10 grams to decay to 5 grams? to 2 grams?

5. The radioactive isotope plutonium 241 decays in a manner satisfying the differential equation \( \frac{dx}{dt} = -0.0525x \), where \( x \) is measured in milligrams and \( t \) in years. Determine the half-life of plutonium 241.

2 Salt Tank Problems

1. A tank starts with a 100 gallons of brine containing a total of 25 pounds of salt. If pure water enters the tank at a rate of 5 gallons per minute and the mixture leaves the tank at the same rate, find the amount of salt in the tank at any future time \( t \).

2. A 500 gallon tank contains a solution of water and salt with the initial amount of salt being 100 pounds. If pure water enters the tank at a rate of 10 gallons per minute and the mixture leaves the tank at the same rate, how long will it take to flush the tank (assume this means that 98% of the original salt is gone)? What would the answer be if we started with \( S \) pounds of salt?
3. A 500 gallon tank contains a solution of water and salt with the initial amount of salt being 100 pounds. If pure water enters the tank at a rate of 8 gallons per minute and the mixture leaves the tank at the same rate, how much salt is in the tank at time $t$?

4. A 500 gallon tank contains a solution of water and salt with the initial amount of salt being 100 pounds. If salt water with a concentration of $1/2$ pound of salt per gallon enters the tank at a rate of 8 gallons per minute and the mixture leaves the tank at the same rate, how much salt is in the tank at time $t$?