1. Let $l = [1, 1], h = [1, -1]$. Let $X = [x_0, x_1, \ldots, x_7]^\top$ be an eight point sample. The single stage filter bank determined by $l$ and $h$ may be implemented computationally by matrix multiplication:

$$X_1 = W_1 X.$$ 

Compute the matrix $W_1$. 
2. Now let $X_2$ be the vector obtained by doing a second stage of a filter bank on the lowpass output of the first stage and keeping the high pass output the same. Compute the matrices $V_2$ and $W_2$ satisfying:

\[ X_2 = V_2 X_1, \]
\[ X_2 = W_2 X. \]
3. Compute $V_3$ and $W_3$ for the third stage. Make sure you draw a diagram.

4. Verify that the pure wave forms defined by the rows of $W_3$ are orthogonal. Draw a bar graph representing the pure wave forms.
5. How can you modify \( l \) and \( h \) to obtain orthonormal pure wave forms.