Notation: A trigonometric interpolating function, with period $L$ and with frequencies up to $k = n$ is a function of the form:

$$ f(t) = a_0 + \sum_{k=1}^{n} a_k \cos\left(\frac{2\pi kt}{L}\right) + \sum_{k=1}^{n} b_k \sin\left(\frac{2\pi kt}{L}\right). $$

We are going to assume $L = 1$ for this entire worksheet. Now consider the data vector $X = \begin{bmatrix} 0 & 1 & -2 & 1 & 0 \end{bmatrix}^t$ that is obtained by sampling an unknown function $g(t)$ every 0.25 second on the interval $[0, 1]$. Samples are taken at both 0 and 1.

1. Write out the equations to interpolate $X$ with a constant sine and cosine terms, for frequencies up to $k = 2$.

2. Eliminate the redundant equations and functions write out the system in matrix form. Then solve the system.
3. Plot the data and the interpolating function on the axes below. (attach a Maple plot if you like)

4. If $f(t)$ has period 1 and is continuous which sample point is redundant? Is the redundant sample value equal to the interpolated value?
Consider the data vector \( X = [ 0 \ 1 \ 1 \ -1 \ -2 ]^t \) that is obtained by sampling a function \( g(t) \) every .02 second on the interval for \([0, 1]\). Note that now we are taking a sample at 0 but not at 1.

5. Set up the matrix to solve for the coefficients for an interpolating polynomial of frequencies to \( k = 2 \). Find the coefficients of the interpolating polynomial.

6. Plot the data and the interpolating function on the axes below. (attach a Maple plot if you like). If the original signal \( g(t) \) has period, 1 what should the next data value be? Does this agree with the interpolating function value? Show this on your plot.
7. A noisy signal is sampled over $[0, 1]$ at $N$ points with a sampling increment of $\Delta t = 1/N$ (it is not sampled at 1). The data is given in `datasamp.mws` on the course webpage. Using the methods above find the major frequencies of the signal. To be specific ignore those frequencies whose coefficients are smaller than 5% of the maximum (in absolute value) coefficient.