Mathematics of Image Processing
Matlab Session #1

Due: Tuesday, September 17

Make sure you make liberal use of help. Keep a diary of your work and attach a cleaned up version of it.

1. Matrices and vectors

1. Set up the following system of equations in matrix form using mathematical notation. Find the solution via Matlab

\[
\begin{align*}
2s + t &= 1 \\
s + 2t + u &= 1 \\
t + 2u + v &= 1 \\
u + 2v + w &= 1 \\
v + 2w + x &= 1 \\
w + 2x + y &= 1 \\
x + 2y + z &= 1 \\
y + 2z &= 1
\end{align*}
\]
2. Scalar Products

2. Are any of the columns of the matrix in question 1 orthogonal? Use Matlab matrix multiplication to get your answer. Write out the mathematical notation for the matrix computation you did in Matlab to answer problem.

3. Do the same for the rows

4. Repeat questions 2 and 3 for the matrices \texttt{Ncossinmat(10)}. Download the script into your working directory.
   
   http://www.rose-hulman.edu/class/ma/broughton/courses/ma439/scripts/Ncossinmat.m
3. Plotting and Aliasing

5. Plot the function \( \sin(5 \cdot 2\pi t) \) at \( N = 128 \) points in \( 0 \leq t \leq 1 \) as done in handouts. (skip \( t = 1 \)). What is the apparent frequency of the sampled signal (cycles per second).

6. Now fill in the following table for \( \sin(n \cdot 2\pi t) \) sampled at \( N \) points.

<table>
<thead>
<tr>
<th>( n )</th>
<th>5</th>
<th>24</th>
<th>69</th>
<th>102</th>
<th>300</th>
<th>5</th>
<th>8</th>
<th>24</th>
<th>60</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>128</td>
<td>128</td>
<td>128</td>
<td>128</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>apparent frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. What conclusions can you draw from Question 5?

4. Noise and distortion

8. Create a simple sum of sines an cosines of varying frequencies for \( 0 \leq t \leq 1 \). Sample at 8192 points and call the vector \( X \). Play the signal with the sound command.

9. Now add some noise \( Y = X + \epsilon N \). The noise vector \( N (N=2*rand(8192,1)-1) \) has its entries uniformly distributed between \(-1\) and \(1\), and the parameter \( \epsilon \) measures the strength of the noise. Compute the maximum distortion (see notes) you can allow and still have reasonable fidelity.

\[
\frac{\|Y - X\|^2}{\|X\|^2} = \frac{\|\epsilon N\|^2}{\|X\|^2} = \epsilon^2 \frac{\|N\|^2}{\|X\|^2}.
\]