Disco II - Quiz 11

Name:__________________________  Box #______________________

1. Rook Polynomials and forbidden positions

You are assigning 4 tasks $T_1, T_2, T_3, T_4$ to five colleagues $C_1, C_2, C_3, C_4, C_5$. Colleague $C_1$ has screwed up $T_4$ in the past and $C_4$ had the same experience with $T_2$. Colleague $C_2$ has threatened to quit if she has to do $T_3$ or $T_4$ again. You really, really need to keep $C_2$.

1. Fill in all the forbidden positions in the following assignment chart.

\[
\begin{array}{cccccc}
   & C_1 & C_2 & C_3 & C_4 & C_5 \\
T_1 &   &   &   &   &   \\
T_2 &   &   & X &   &   \\
T_3 & X &   &   &   &   \\
T_4 & X & X &   &   &   \\
\end{array}
\]

2. Compute the rook polynomial for the chessboard consisting of the forbidden positions.

$FP =$ forbidden positions.

\[
r(FP) = r\left(\begin{array}{cc}
X & X \\
X & X \\
\end{array}\right) r\left(\begin{array}{c}
X \\
X \\
\end{array}\right)
\]

\[
= \left(r\left(\begin{array}{cc}
X & X \\
X & X \\
\end{array}\right) + xr\left(\begin{array}{c}
X \\
X \\
\end{array}\right)\right) r\left(\begin{array}{c}
X \\
X \\
\end{array}\right)
\]

\[
= (1 + 2x + x(1 + x))(1 + x)
\]

\[
= 1 + 4x + 4x^2 + x^3.
\]

Therefore

\[
r_1 = 4, \ r_2 = 4, \ r_3 = 1, \ r_4 = 0
\]

3. How many ways can you assign the tasks to your colleagues, following the constraints?
$c_i$: Task $T_i$ is assigned to a forbidden colleague:

\[
\begin{align*}
S_0 &= 5 \cdot 4 \cdot 3 \cdot 2 = 120 \\
S_1 &= r_1 \cdot 4 \cdot 3 \cdot 2 = 96 \\
S_2 &= r_2 \cdot 3 \cdot 2 = 24 \\
S_3 &= r_3 \cdot 2 = 2 \\
S_4 &= r_4 = 0
\end{align*}
\]

\[E_0 = 120 - 96 + 24 - 2 + 0 = 46\]