Signal Processing First

Lecture 17
IIR Filters: $H(z)$ and Frequency Response

LECTURE OBJECTIVES
- SYSTEM FUNCTION: $H(z)$
- $H(z)$ has POLES and ZEROS
- FREQUENCY RESPONSE of IIR
  - Get $H(z)$ first
  \[ H(e^{j\omega}) = H(z) \bigg|_{z = e^{j\omega}} \]
- THREE-DOMAIN APPROACH

THREE DOMAINS

$H(z) = \text{z-Transform}\{ h[n] \}$

- FIRST-ORDER IIR FILTER:
  \[ y[n] = a_1 y[n-1] + b_0 x[n] \]
  \[ h[n] = b_0 (a_1)^n u[n] \]
  \[ H(z) = \frac{b_0}{1 - a_1 z^{-1}} \]

First-Order Transform Pair

\[ h[n] = b a^n u[n] \leftrightarrow H(z) = \frac{b}{1 - a z^{-1}} \]

DELAY PROPERTY of $X(z)$

- DELAY in TIME $\leftrightarrow$ Multiply $X(z)$ by $z^{-1}$
  \[ x[n] \leftrightarrow X(z) \]
  \[ x[n-1] \leftrightarrow z^{-1} X(z) \]
**Z-Transform of IIR Filter**

- DERIVE the SYSTEM FUNCTION $H(z)$
- Use **DELAY** PROPERTY
  \[ y[n] = a_1y[n-1] + b_0x[n] + b_1x[n-1] \]

**SYSTEM FUNCTION of IIR**

- NOTE the FILTER COEFFICIENTS

**SYSTEM FUNCTION**

- Given: DIFFERENCE EQUATION:
  \[ y[n] = 0.8y[n-1] + 3x[n] - 2x[n-1] \]
- READ the FILTER COEFFS:

**POLES & ZEROS**

- Find the Poles and Zeros
- ROOTS of Numerator & Denominator
  \[ H(z) = \frac{b_0 + b_1z^{-1}}{1 - a_1z^{-1}} \]

**EXAMPLE: Poles & Zeros**

- VALUE of $H(z)$ at POLES is **INFINITE**
  \[ H(z) = \frac{2 + 2z^{-1}}{1 - 0.8z^{-1}} \]
  \[ H(z) = \frac{2 + 2(-1)}{1 - 0.8(-1)} = 0 \]
  \[ H(z) = \frac{2 + 2(0.8)^{-1}}{1 - 0.8(0.8)^{-1}} = 0 \rightarrow \infty \]

**POLE-ZERO PLOT**

- ZERO at $z = -1$
- POLE at $z = 0.8$
**FREQUENCY RESPONSE**

- SYSTEM FUNCTION: H(z)
- H(z) has DENOMINATOR
- FREQUENCY RESPONSE of IIR
  - We have H(z)
    \[ H(e^{j\omega}) = H(z) \big|_{z = e^{j\omega}} \]
- THREE-DOMAIN APPROACH
  \[ h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\omega}) \]

**FREQUENCY RESPONSE**

- EVALUATE on the UNIT CIRCLE
  \[ H(e^{j\omega}) = H(z) \big|_{z = e^{j\omega}} \]
  \[ H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} \]
  \[ H(e^{j\hat{\omega}}) = H(z) \big|_{z = e^{j\hat{\omega}}} = \frac{b_0 + b_1 e^{-j\hat{\omega}}}{1 - a_1 e^{-j\hat{\omega}}} \]

**UNIT CIRCLE**

- MAPPING BETWEEN \( z \) and \( \hat{\omega} \)
  - \( z = e^{j\omega} \)
  - \( z = 1 \leftrightarrow \hat{\omega} = 0 \)
  - \( z = -1 \leftrightarrow \hat{\omega} = \pm \pi \)
  - \( z = \pm j \leftrightarrow \hat{\omega} = \pm \frac{1}{2} \pi \)

**MOVIE for H(z) in 3-D**

- POLES to H(z) to Frequency Response
- TWO POLES SHOWN
**Frequency Response from H(z)**

Walking around the Unit Circle

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**3 DOMAINS MOVIE: IIR**

PeZ Demo: Pole-Zero Placing