Signal Processing First

Lecture 11
Linearity & Time-Invariance Convolution

LECTURE OBJECTIVES
- GENERAL PROPERTIES of FILTERS
  - LINEARITY
  - TIME-INVARINCE
  - \( \implies \) CONVOLUTION
- BLOCK DIAGRAM REPRESENTATION
  - Components for Hardware
  - Connect Simple Filters Together to Build More Complicated Systems

OVERVIEW
- IMPULSE RESPONSE, \( h[n] \)
- FIR case: same as \( \{b_k\} \)
- CONVOLUTION
  - GENERAL: \( y[n] = h[n] \ast x[n] \)
  - GENERAL CLASS of SYSTEMS
  - LINEAR and TIME-INVARIANT
  - ALL LTI systems have \( h[n] \) & use convolution

BUILDING BLOCKS
- BUILD UP COMPLICATED FILTERS
  - FROM SIMPLE MODULES
  - Ex: FILTER MODULE MIGHT BE 3-pt FIR

DIGITAL FILTERING
- CONCENTRATE on the FILTER (DSP)
- DISCRETE-TIMESIGNALS
  - FUNCTIONS of \( n \), the “time index”
  - INPUT \( x[n] \)
  - OUTPUT \( y[n] \)

GENERAL FIR FILTER
- FILTER COEFFICIENTS \( \{b_k\} \)
- DEFINE THE FILTER
  \[
  y[n] = \sum_{k=0}^{M} b_k x[n - k] 
  \]
  
  For example, \( b_k = \{3, -1, 2, 1\} \)
  \[
  y[n] = \sum_{k=0}^{3} b_k x[n - k] = 3x[n] - x[n-1] + 2x[n-2] + x[n-3] 
  \]
MATLAB for FIR FILTER

- \( y = \text{conv}(bb, xx) \)
  - VECTOR \( bb \) contains Filter Coefficients
  - SP-First: \( y = \text{firfilt}(bb, xx) \)
- FILTER COEFFICIENTS \( \{b_k\} \)

\[
y[n] = \sum_{k=0}^{M} b_k x[n-k]
\]

SPECIAL INPUT SIGNALS

- \( x[n] = \text{SINUSOID} \)
- Frequency Response
- \( x[n] \) has only one NON-ZERO VALUE

UNIT-IMPULSE

FILTER COEFFICIENTS \( \{b_k\} \)

\[
\sum_{k=0}^{M} b_k x[n-k]
\]

FIR IMPULSE RESPONSE

- Convolution = Filter Definition
- Filter Coefs = Impulse Response

| \( n \) | \( n < 0 \) | 0 | 1 | 2 | 3 | \ldots | \( M \) | \( M+1 \) | \( n > M+1 \) |
|---|---|---|---|---|---|---|---|---|
| \( x[n] = \delta[n] \) | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( y[n] = h[n] \) | 0 | \( b_0 \) | \( b_1 \) | \( b_2 \) | \( b_3 \) | \ldots | \( b_M \) | 0 | 0 |

\[
h[n] = \sum_{k=0}^{M} b_k \delta[n-k]
\]

MATH FORMULA for \( h[n] \)

- Use SHIFTED IMPULSES to write \( h[n] \)

\[
h[n] = 1 \quad \delta[n] \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
\]

CONVOLUTION Example

\[
h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]
\]

\[
x[n] = u[n]
\]

LTI: Convolution Sum

- Output = Convolution of \( x[n] \) & \( h[n] \)
  - NOTATION: \( y[n] = h[n] * x[n] \)
  - Here is the FIR case:

\[
y[n] = \sum_{k=0}^{M} h[k] x[n-k]
\]

Same as \( b_k \).
**Turn to Your Neighbor**

- **FIR Filter** is “FIRST DIFFERENCE”
  - $y[n] = x[n] - x[n-1]$  
- **INPUT** is “UNIT STEP”
  - $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$
- Find $y[n]$

**HARDWARE STRUCTURES**

- **INTERNAL STRUCTURE** of “FILTER”
  - **WHAT COMPONENTS ARE NEEDED?**
  - **HOW DO WE “HOOK” THEM TOGETHER?**
  - **SIGNAL FLOW GRAPH NOTATION**

**HARDWAREATOMS**

- **Add, Multiply & Store**
  - $y[n] = \sum_{k=0}^{M} b_k x[n-k]$  

**FIR STRUCTURE**

- **Direct Form**
  - $y[n] = \sum_{k=0}^{M} b_k x[n-k]$  

**Moore’s Law for TI DSPs**

- Double every 18 months?
SYSTEM PROPERTIES

\[ x[n] \rightarrow \text{SYSTEM} \rightarrow y[n] \]

- MATHEMATICAL DESCRIPTION
- TIME-INVARIANCE
- LINEARITY
- CAUSALITY
  - “No output prior to input”

TIME-INVARIANCE

- IDEA:
  - “Time-Shifting the input will cause the same time-shift in the output”

- EQUIVALENTLY,
  - We can prove that
    - The time origin (n=0) is picked arbitrary

TESTING Time-Invariance

- IDEA:
  - Time-Shifting the input will cause the same time-shift in the output

  EQUIVALENTLY,
  - We can prove that
    - The time origin (n=0) is picked arbitrary

LINEARITY

- LINEARITY = Two Properties
- SCALING
  - “Doubling x[n] will double y[n]”

- SUPERPOSITION:
  - “Adding two inputs gives an output that is the sum of the individual outputs”

TESTING LINEARITY

LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - IMPULSE RESPONSE \( h[n] \)
  - CONVOLUTION: \( y[n] = x[n] * h[n] \)
    - The “rule” defining the system can ALWAYS be rewritten as convolution
  - FIR Example: \( h[n] \) is same as \( b_k \)
**Turn to Your Neighbor**

- FIR Filter is “FIRST DIFFERENCE”
  - \( y[n] = x[n] - x[n-1] \)
- Write output as a convolution
  - Need impulse response

- Then, another way to compute the output:

**CASCADE SYSTEMS**

- Does the order of \( S_1 \) & \( S_2 \) matter?
  - NO, LTI SYSTEMS can be rearranged!!!
- WHAT ARE THE FILTER COEFS? \( \{ b_k \} \)

**CASCADE EQUIVALENT**

- Find “overall” \( h[n] \) for a cascade?

**One View of DSP, c. 1976**

- “That discipline which has allowed us to replace a circuit previously composed of a capacitor and a resistor with two anti-aliasing filters, an A-to-D and a D-to-A converter, and a general purpose computer (or array processor) so long as the signal we are interested in does not vary too quickly.”

  Thomas P. Barnwell, III