In this Lab you will first obtain a sixth order model of a three degree of freedom spring/mass/damper system (all three carts can move), design a state variable feedback controller for it, implement the controller on the real system, and then compare the predicted response with the actual response. You will probably make at least two observations during this lab:

- models are not perfect, they are just guides
- real motors have real limits, which can make designing more difficult

Pre-Lab

1) Print out this lab and read it.

Consider the following model of the three DOF system we will be using.

2) Draw freebody diagrams of the forces on the three masses.

3) The equations of motion for the three masses can be written

\[ m_1 \ddot{x}_1 + c_1 \dot{x}_1 + (k_1 + k_2)x_1 = F + k_2x_2 \]  
\[ m_2 \ddot{x}_2 + c_2 \dot{x}_2 + (k_2 + k_3)x_2 = k_2x_1 + k_3x_3 \]  
\[ m_3 \ddot{x}_3 + c_3 \dot{x}_3 + (k_3 + k_4)x_3 = k_3x_2 \]

If we define \( q_1 = x_1, q_2 = \dot{x}_1, q_3 = x_2, q_4 = \dot{x}_2, q_5 = x_3, \) and \( q_6 = \dot{x}_3 \) show that we get the following state equations

\[
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3 \\
\dot{q}_4 \\
\dot{q}_5 \\
\dot{q}_6
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-(k_1+k_2)/m_1 & -(c_1/m_1) & (k_2/m_1) & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
(k_2/m_2) & 0 & -(k_2+k_3)/m_2 & -(c_2/m_2) & (k_3/m_2) & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & (k_3/m_3) & 0 & -(k_3+k_4)/m_3 & -(c_3/m_3)
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2 \\
q_3 \\
q_4 \\
q_5 \\
q_6
\end{bmatrix} +
\begin{bmatrix}
0 \\
1/m_1 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
F
\]
In order to get the $A$ and $B$ matrices, we need to determine all of the quantities in the above matrices.

4) Now we want to rewrite equations 1, 2, and 3 in terms of $\zeta_1$, $\omega_1$, $\zeta_2$, $\omega_2$, $\zeta_3$ and $\omega_3$ as

\[
\ddot{x}_1 + 2\zeta_1\omega_1\dot{x}_1 + \omega_1^2 x_1 = \frac{k_2}{m_1} x_2 + \frac{1}{m_1} F 
\]  
(4)

\[
\ddot{x}_2 + 2\zeta_2\omega_2\dot{x}_2 + \omega_2^2 x_2 = \frac{k_2}{m_2} x_1 + \frac{k_3}{m_2} x_3 
\]  
(5)

\[
\ddot{x}_3 + 2\zeta_3\omega_3\dot{x}_3 + \omega_3^2 x_3 = \frac{k_3}{m_3} x_2 
\]  
(6)

We will get our initial estimates of the parameters $\zeta_1$, $\omega_1$, $\zeta_2$, $\omega_2$, $\zeta_3$ and $\omega_3$ using the log-decrement method. Assuming we measure these parameters, show how $A_{2,1}$, $A_{2,2}$, $A_{4,3}$, $A_{4,4}$, $A_{6,5}$ and $A_{6,6}$ can be determined.

5) By taking the Laplace transforms of equations 4, 5 and 6 (assuming zero initial condition), we get the following

\[
\frac{X_3(s)}{F(s)} = \frac{1}{m_1 \, m_2 \, m_3} \frac{k_2 \, k_3}{\Delta(s)} 
\]

where

\[
\Delta(s) = (s^2 + 2\zeta_1\omega_1 s + \omega_1^2)(s^2 + 2\zeta_2\omega_2 s + \omega_2^2)(s^2 + 2\zeta_3\omega_3 s + \omega_3^2) 
\]  
(8)

\[
-\frac{k_2}{m_1} \frac{k_2}{m_2} (s^2 + 2\zeta_3\omega_3 s + \omega_3^2) 
\]  
(9)

\[
-\frac{k_3}{m_2} \frac{k_3}{m_3} (s^2 + 2\zeta_1\omega_1 s + \omega_1^2) 
\]  
(10)

We would really rather write

\[
\Delta(s) = (s^2 + 2\zeta_a\omega_a s + \omega_a^2)(s^2 + 2\zeta_b\omega_b s + \omega_b^2)(s^2 + 2\zeta_c\omega_c s + \omega_c^2) 
\]  
(11)

Now we need to equate coefficients between powers of $s$ in these two expressions for $\Delta(s)$. For $s^6$ we get

\[
1 = 1 
\]

For $s^5$ we get

\[
2\zeta_a\omega_a + 2\zeta_b\omega_b + 2\zeta_c\omega_c = 2\zeta_1\omega_1 + 2\zeta_2\omega_2 + 2\zeta_3\omega_3 
\]

For $s^4$ we get

\[
\omega_a^2 + \omega_b^2 + \omega_c^2 + 4\zeta_a\omega_a\zeta_b\omega_b + 4\zeta_a\omega_a\zeta_c\omega_c + 4\zeta_b\omega_b\zeta_c\omega_c = \omega_1^2 + \omega_2^2 + \omega_3^2 + 4\zeta_1\omega_1\zeta_2\omega_2 + 4\zeta_1\omega_1\zeta_3\omega_3 + 4\zeta_2\omega_2\zeta_3\omega_3 
\]

For $s^3$ we get

\[
\zeta_a\omega_a\omega_b^2 + \zeta_a\omega_a\omega_c^2 + \zeta_b\omega_b\omega_a^2 + \zeta_b\omega_b\omega_c^2 + \zeta_c\omega_c\omega_a^2 + \zeta_c\omega_c\omega_b^2 + 4\zeta_a\omega_a\zeta_b\omega_b\zeta_c\omega_c 
\]  
\[
= \zeta_1\omega_1\omega_2^2 + \zeta_1\omega_1\omega_3^2 + \zeta_2\omega_2\omega_1^2 + \zeta_2\omega_2\omega_3^2 + \zeta_3\omega_3\omega_1^2 + \zeta_3\omega_3\omega_2^2 + 4\zeta_1\omega_1\zeta_2\omega_2\zeta_3\omega_3 
\]
These equations will be used to actually determine the values of $\zeta_1$, $\omega_1$. You need to write out the equations for the coefficients of $s^2$ (6 terms and 8 terms), $s$ (3 terms and 5 terms), and $s^0$ (1 term and 3 terms).

6) It is more convenient to write this transfer function as

$$\frac{X_3(s)}{F(s)} = \frac{K_3}{\left(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s + 1\right)\left(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1\right)\left(\frac{1}{\omega_c^2}s^2 + \frac{2\zeta_c}{\omega_c}s + 1\right)}$$

(12)

What is $K_3$ in terms of $k_i$, $m_i$, and $\omega_a$, $\omega_b$, and $\omega_c$?

7) Using equations 6 and 7, show that we can write the transfer function for $X_2(s)$ as

$$\frac{X_2(s)}{F(s)} = \frac{\frac{1}{m_1} m_2 (s^2 + 2\zeta_3 \omega_3 s + \omega_3^2)}{\Delta(s)}$$

(13)

8) This is more convenient to write as

$$\frac{X_2(s)}{F(s)} = \frac{K_2 \left(\frac{1}{\omega_3^2}s^2 + \frac{2\zeta_3}{\omega_3}s + 1\right)}{\left(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s + 1\right)\left(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1\right)\left(\frac{1}{\omega_c^2}s^2 + \frac{2\zeta_c}{\omega_c}s + 1\right)}$$

(14)

What is $K_2$ in terms of $k_i$, $m_i$, and $\omega_a$, $\omega_b$, $\omega_c$ and $\omega_3$?

9) Show that

$$A_{6,3} = \frac{k_3}{m_3} = \frac{K_3}{K_2} \omega_3^2$$

(15)

10) Using equation 5, 7, and 13, show that we can write the transfer function for $X_1(s)$ as

$$\frac{X_1(s)}{F(s)} = \frac{\frac{1}{m_1} \left[(s^2 + 2\zeta_2 \omega_2 s + \omega_2^2)(s^2 + 2\zeta_3 \omega_3 s + \omega_3^2) - \frac{k_3}{k_1} \frac{k_1}{m_3}ight]}{\Delta(s)}$$

(16)

11) We would rather write this as

$$\frac{X_1(s)}{F(s)} = \frac{\frac{1}{m_1} \left[(s^2 + 2\zeta_x \omega_x s + \omega_x^2)(s^2 + 2\zeta_y \omega_y s + \omega_y^2)\right]}{\Delta(s)}$$

(17)

Again we need to equation the coefficients of equal powers of $s$ in the numerator.

For $s^4$ we get

$$1 = 1$$

For $s^3$ we get

$$\zeta_x \omega_x + \zeta_y \omega_y = \zeta_2 \omega_2 + \zeta_3 \omega_3$$
For $s^2$ we get

$$\omega_x^2 + \omega_y^2 + 4\zeta_x \omega_x \omega_y = \omega_2^2 + \omega_3^2 + 4\zeta_2 \omega_x \omega_y$$

For $s$ we get

$$\zeta_x \omega_x \omega_y^2 + \zeta_y \omega_y \omega_x^2 = \zeta_2 \omega_2 \omega_x^2 + \zeta_3 \omega_3 \omega_y^2$$

We will use these to determine $\zeta_2$ and $\omega_2$. Show that by equating the coefficients for $s^0$ we arrive at the relationship

$$A_{4,5} = \frac{k_2}{m_2} = \omega_2^2 - \omega_3^2 \frac{\omega_y^2}{A_{6,3}}$$

12) A more convenient way to write this transfer function is

$$\frac{X_1(s)}{F(s)} = \frac{K_1 \left[ \left( \frac{1}{\omega_x^2} s^2 + \frac{2\zeta_x}{\omega_x} s + 1 \right) \left( \frac{1}{\omega_y^2} s^2 + \frac{2\zeta_y}{\omega_y} s + 1 \right) \right]}{\left( \frac{1}{\omega_a^2} s^2 + \frac{2\zeta_a}{\omega_a} s + 1 \right) \left( \frac{1}{\omega_b^2} s^2 + \frac{2\zeta_b}{\omega_b} s + 1 \right) \left( \frac{1}{\omega_c^2} s^2 + \frac{2\zeta_c}{\omega_c} s + 1 \right)}$$

(19)

What is $K_1$ in terms of $k_i, m_i, \omega_a, \omega_b, \omega_c, \omega_x$, and $\omega_y$?

13) Show that

$$A_{4,1} = \frac{k_2}{m_2} = \frac{K_2}{K_1} \frac{\omega_y^2}{\omega_3^2}$$

(20)

14) By using the results from problem 5, for the $s^0$ terms, show that one way to obtain an estimate for $\frac{k_2}{m_1}$ is

$$A_{2,3} = \frac{k_2}{m_1} = \frac{\omega_1^2 \omega_2^2 \omega_3^2 - \omega_2^2 \omega_3^2 \omega_1^2 - A_{4,5} A_{6,3} \omega_1^2}{\omega_2^2 A_{4,1}}$$

(21)

15) All that’s left is to find $\frac{1}{m_1}$, which is $b_2$. It’s important to understand that this parameter also includes “scaling” on $F(s)$. Now assume we look at the closed loop response to a simple proportional type controller. Hence we have the system shown below:

For a step response of amplitude $Amp$, show that the steady state value of $x_3(t), x_{3,ss}$ is

$$x_{3,ss} = \frac{K_3 k_p Amp}{1 + K_3 k_p}$$

(22)
and that by rearranging this equation we get

$$b_2 = \frac{1}{m_1} = \frac{x_{3,ss}}{k_p(Amp - x_{3,ss})} \frac{\omega_n^2 \omega_b^2 \omega_c^2}{A_{4,1}A_{6,3}}$$  \hspace{1cm} (23)$$

### Summary

1) Fit frequency response data to

$$\frac{X_3(s)}{F(s)} = \frac{K_3}{(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s + 1)(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1)(\frac{1}{\omega_c^2}s^2 + \frac{2\zeta_c}{\omega_c}s + 1)}$$  \hspace{1cm} (24)$$

this will give us initial estimates for $K_3, \zeta_a, \omega_a, \zeta_b, \omega_b, \zeta_c, \text{and} \omega_c$.

2) Using the above parameters as initial estimates, fit frequency response data to

$$\frac{X_2(s)}{F(s)} = \frac{K_2(\frac{1}{\omega_3^2}s^2 + \frac{2\zeta_3}{\omega_3}s + 1)}{(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s + 1)(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1)(\frac{1}{\omega_c^2}s^2 + \frac{2\zeta_c}{\omega_c}s + 1)}$$  \hspace{1cm} (25)$$

this will give us estimates for $K_2, \zeta_3, \omega_3$, and new estimates (they will not be very different from the old estimates) for $\zeta_a, \omega_a, \zeta_b, \omega_b, \zeta_c \text{ and } \omega_c$.

3) Using the above parameters as initial estimates, fit frequency data to

$$\frac{X_1(s)}{F(s)} = \frac{K_1 \left[ (\frac{1}{\omega_x^2}s^2 + \frac{2\zeta_x}{\omega_x}s + 1)(\frac{1}{\omega_y^2}s^2 + \frac{2\zeta_y}{\omega_y}s + 1) \right]}{(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s + 1)(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1)(\frac{1}{\omega_c^2}s^2 + \frac{2\zeta_c}{\omega_c}s + 1)}$$  \hspace{1cm} (26)$$

4) Using the relationships derived in part 11, find the values of $\zeta_2$ and $\omega_2$, then using the relationships in part 5, find values for $\zeta_1$ and $\omega_1$.

5) Estimate all of the parameters in $A$.

6) Look at the step response to estimate $b_2$. 
We need to first **identify** the system:

1. For this lab, be sure
   - The spring between the first cart and the motor is a stiff spring
   - The mass on the first cart is greater than or equal to the mass on the second cart, and the mass on the second cart has mass greater than or equal to the mass on the third cart.

2. Estimate an initial second order system model using time domain analysis for each mass (cart). Only the cart you are analyzing should be free to move (lock the other carts in place). Use the `log_dec` program to get initial estimates of $\zeta_1$, $\omega_1$, $\zeta_2$, $\omega_2$, $\zeta_3$ and $\omega_3$.

3. Be sure all carts are free to move. Measure the frequency response at increments of 0.25 Hz, starting at 0.5 Hz and going up to approximately 7 Hz. Be sure you identify three resonant peaks for the third cart. This may mean you need to go beyond 7 Hz. **You need to measure the amplitudes (in counts) of all three of the carts.** Be very careful here, if you plot two of them on the same graph there are two different scales (one on the left axis, on on the right).

4. Enter the frequency data into the program `process_data_3cars`, and then run the program with the output going to variable `data`, i.e., type
   ```
   > data = process_data_3cars;
   ```

5. Now use the program `model_3cars` The arguments to this procedure are the initial guesses of the final parameters. The arguments (and reasonable guesses) are:
   - `data`, data from the `process_data_3cars` routine
   - $K_3$, the estimated gain, try 1000 for an initial guess
   - $\omega_a$, use the estimated first resonant frequency in rad/sec
   - $\zeta_a$, use 0.1
   - $\omega_b$, use the estimated second resonant frequency in rad/sec
   - $\zeta_b$, use 0.1
   - $\omega_c$, use the estimated third resonant frequency in rad/sec
   - $\zeta_c$, use 0.1
   - $\omega_1$, use your results from `log_dec`
   - $\zeta_1$, use your results from `log_dec`
   - $\omega_2$, use your results from `log_dec`
   - $\zeta_2$, use your results from `log_dec`
   - $\omega_3$, use your results from `log_dec`
   - $\zeta_3$, use your results from `log_dec`

The output of the routine is the $A$ matrix, so you should invoke the routine with
> A = model_3carts(...

6. You should get four plots from this routine, the first is the estimated transfer function for $X_3$, the second is the estimated transfer function for $X_2$, the third is the estimated transfer function for $X_1$. The last plot is identical to that for $X_1$ but it includes the final estimates for $\omega_1$, $\zeta_1$, $\omega_2$, $\zeta_2$, $\omega_3$, and $\zeta_3$. You need to include the first three plots in your memo, so save them now! At this point you will may need to add more data points. In particular, you need to be sure there are at least 4 data points near each peak and 4 data points near each null. Most likely you will need to add data points for the nulls of $X_2$ and $X_1$. Edit `process_data_3carts` and rerun it, then rerun `model_3carts`.

7. Before you go on, make sure I see the fit of your data to the measured frequency response. You need to include these three final plots in your memo, so save them now!

8. Finally, we need to estimate $b_2$, to be described below.

**Estimating $b_2$**

0. Set the units

Click **Setup → User Units** and set the units to **cm**.

1. Setting up the controller

Click **Setup → Control Algorithm**. Be sure the system is set for *Continuous Time*. Select **PID** under **Control Algorithm**. Click on **Setup Algorithm**. Be sure **Feedback** is from **Encoder 3**. Set $k_p$ to a small number (less than or equal to 0.05) and be sure $k_d = 0$ and $k_i = 0$. Then click **OK**. Next Click **Implement Algorithm**. The click **OK**.

2. Setting up the closed loop trajectory

Click **Command → Trajectory**. Select **Step** and click on **Setup**. Select **Closed Loop Step** and set **Step Size** to 0.5 to 1.5 cm. Be sure to record this step size (we’ll refer to the amplitude as $Amp$ below). Set the **Dwell Time** to something like 2000 ms, this is the time the system will be recording data. Finally click **OK**, then **OK** and you should be back to the main menu.

3. Executing the closed loop step

Click **Command Execute**. A menu box will come up with a number of options, and a big green **Run** button. Click on the **Run** button. When the system has finished collecting data, a box will appear indicating the how many sample points of data have been collected. (If you have hit a stop, the system stops recording data. This usually means you’re input amplitude was too large or $k_p$ was too large. ) Click on **OK** to get back to the main menu.
4. Determining the steady state value

Click Plotting → Setup Plot, or just Plotting Data → Plot Data. Look at the steady state value \( x_{3,ss} \). You may need to change the dwell time if your system has not reached steady state.

5. Estimating \( b_2 \)

Estimate \( b_2 \) using the formula derived previously:

\[
 b_2 = \frac{1}{m_1} = \frac{x_{3,ss}}{k_p(Amp - x_{3,ss})} \frac{\omega_n^2 \omega_f^2 \omega_c^2}{A_{4,1}A_{6,3}} \tag{27}
\]

Note that the value \( \frac{\omega_n^2 \omega_f^2 \omega_c^2}{A_{4,1}A_{6,3}} \) is printed out by model_3carts and is referred to as the variable scale.

You need to go through this procedure at least three times for each configuration. You must use at least two different values of \( k_p \) and two different values of input amplitude \( Amp \). If none of the steady state values is larger than 0.4 cm, increase either \( k_p \) or \( Amp \). Average the three results to get your \( b_2 \) (they should be similar). For the trials I’ve run, I’ve got \( b_2 \) between 6,000 and 12,000. Your’s may be outside this range though.

Designing and Implementing the State variable Controllers

Our general goals for the state variable controllers are as follows:

- produce a position error of less than 0.15
- reach steady state within 1 seconds (the faster the better)
- have as little overshoot as you can manage

Here are some general ideas:

- You need to try and use positive values for \( k_1, ..., k_6 \) (\( k = [k_1 \ k_2 \ k_3 \ k_4 \ k_5 \ k_6] \)). The system does not respond very well to negative values, though negative values with a small magnitude may work OK. In particular, the steady state values may be off.
- \( k_1 \) should be larger than \( k_2 \), \( k_3 \) should be larger than \( k_4 \), and \( k_5 \) larger than \( k_6 \). \( k_2, k_4, \) and \( k_6 \) are multiplying the derivatives, and estimates of derivatives tends to be noisy.
- Try to keep \( k_2, k_4, \) and \( k_6 \) less than 0.05, and \( k_1, k_3, \) and \( k_5 \) less than 1.0.
- You may need to reset the controller often, such as every time you want to implement a new controller. Click Utility → Reset Controller. Only do this before you have implemented a controller.
- You may need to rephase the motor. Click Utility → Rephase Motor
- Be sure to Implement the controller you have designed.
- Try and track a step with an input amplitude of 0.5 to 1 cm.
0. Determine Which Cart Location is the Output

You will need to determine which cart location is the output of the system. This will allow you to construct the appropriate $C$ matrix. Most likely you’ll want the position of cart 3 to be the output, but you may want to try controlling the position of cart 1 and cart 2 also.

1. Estimate the Gains

Use the program `state_variables_3carts` to guess values for $k_1...k_6$. The program will print out the corresponding locations of the closed loop poles and the correct gain $k_{pf}$, as well as produce a plot of the system with state variable feedback.

The arguments to this program are:

- the amplitude of the input signal (in cm)
- the $A$ matrix
- the $B$ vector
- the $C$ vector
- the feedback gain matrix $k = [k_1 \ k_2 \ k_3 \ k_4 \ k_5 \ k_6]$
- the length of time to run the simulation for
- the file name with containing the response of the real system in single quotes. At this point, the filename is just ”

2. Implement the Gains on the ECP System

Once your simulated system has a reasonable response, and probably more importantly, reasonable gains, try running the ECP system with these gains. If the gains are not too large and the system works, save the results to a file. If the system buzzes and doesn’t work, go back to step 1 and try again.

3. Comparing the Simulation and the ECP system

Edit the file you saved in part 2 so Matlab can read it. Run the program `state_variable_3carts` again, with the same gains as you used on the system. This time the last argument to the program is the name of the file you saved the response of the system into. You should get a plot containing both the real system and the simulated system. You may want to reduce the final time of the plot so there is not alot of time at steady state showing.
4. Practice Makes Perfect

Try at least three different combinations of gains before you move on to the next method. Be sure to produce a plot for each system, and record the gains and closed loop poles for each system.

**Linear Quadratic Regulator Method**

1. Estimating the Feedback Gains

Use the Matlab routine `lqr` to estimate the feedback gains $k_1, \ldots, k_6$. The arguments to this routine are

- the $A$ matrix of the system
- the $B$ matrix of the system
- a penalty matrix $Q$
- a penalty matrix $R$

(Note there is one more possible argument, but we won’t use it. Type `help lqr` for more information).

The Linear Quadratic Regulator finds the gain $K$ to minimize

$$ J = \int_0^\infty \left[ x^T(t)Qx(t) + u(t)Ru(t) \right] dt $$

where

$$ \dot{x}(t) = Ax(t) + Bu(t) $$

$$ u(t) = -k x(t) $$

In our case $Q$ is a six by six positive definite matrix, and $R$ is a scalar. Since $Q$ is most likely a diagonal matrix, it’s easiest to iterate using the following command in Matlab

```matlab
> K = lqr(A,B,diag([q1 q2 q3 q4 q5 q6]),R)
```

where $q1, q2, q3, q4, q5$ and $q6$ are the desired diagonal elements of $Q$ and $R$ is a scalar. In general, as $R$ gets larger (it may have to get very large), the size of the gains goes down. Also, if a gain is negative, try setting the weight on that gain very large. Note that if all of the gains are large there is no real effect! Iterate on values of $Q$ and $R$ until you think you have something that works.

2. Determining $k_{pf}$

Again used the program `state_variables_3carts`. The values for $K$ have been determined by the lqr routine above. The program will print out the corresponding locations of the closed loop poles and the correct gain $k_{pf}$, as well as produce a plot of the system with state variable feedback.

The arguments to this program are:
• the amplitude of the input signal (in cm)
• the $A$ matrix
• the $B$ matrix
• the $C$ matrix
• the feedback gain matrix $k = [k_1, k_2, k_3, k_4, k_5, k_6]$
• the length of time to run the simulation for
• the file name with containing the response of the real system in single quotes. At this point, the filename is just 

3. Implement the Gains on the ECP System

Once your simulated system has a reasonable response, and probably more importantly, reasonable gains, try running the ECP system with these gains. If the gains are not too large and the system works, save the results to a file. If the system buzzes and doesn’t work, go back to step 1 and try again.

4. Comparing the Simulation and the ECP system

Edit the file you saved in part 3 so Matlab can read it. Run the program `state_variables_3carts` again, with the same gains as you used on the system. This time the last argument to the program is the name of the file you saved the response of the system into. You should get a plot containing both the real system and the simulated system. You may want to reduce the final time of the plot so there is not alot of time at steady state showing.

4. Practice Makes Perfect

Try at least three different combinations of gains (corresponding to three different values of $Q$ and $R$). Be sure to produce a plot for each system, and record the gains and closed loop poles for each system.
Your memo should compare (briefly) the response of the model and the response of the real system for the different gains you tried. You should have some description of the configuration of the system you were trying to control.

You should include the following items as attachments. Most of these are figures which should have reasonable captions.

- The initial fit of the transfer functions to the measured frequency response of the system.
- The final fit of the transfer functions to the measured frequency response of the system.
- The data used to determine $b_2$.
- The final model of the system.
- The predicted and actual response of the system to each of the different controllers where you guessed the values of $k$, and the corresponding closed loop pole locations.
- The predicted and actual response of the system to each of the different controllers where you used the lqr algorithm to determine the values of $k$. Also record values of $Q$ and $R$ used and the corresponding closed loop pole locations.