1) For a system with plant

\[ G_p(s) = \frac{s + 3}{s(s - 1)} \]

show that the quadratic optimal closed loop transfer function is

\[ G_0(s) = \frac{10(s + 3)}{s^2 + 12.7s + 30} \]

when \( q = 100 \).

2) For a system with plant

\[ G_p(s) = \frac{s - 1}{s(s - 2)} \]

show that the quadratic optimal closed loop transfer function is

\[ G_0(s) = \frac{-10(s - 1)}{s^2 + 11.1s + 10} \]

when \( q = 100 \).

3) For a one degree of freedom system like we have in lab, with plant

\[ G_p(s) = \frac{15}{0.0025s^2 + 0.0080s + 1} \]

a) Show that when \( q = 0.1 \) the quadratic optimal closed loop transfer function is

\[ G_0(s) = \frac{1856.6}{s^2 + 55.5s + 1939.1} \]

b) Show that the controller is given by

\[ G_c(s) = \frac{0.0038s^2 + 0.012s + 1.5}{0.012s^2 + 0.67s + 1} \]
For the following three problems, it is useful to remember

- our compensator has the form \( G_c(s) = \frac{B(s)}{A(s)} \), where \( B(s) = B_0 + B_1s + ... \) and \( A(s) = A_0 + A_1s + ... \)
- the plant has the form \( G_p(s) = \frac{N(s)}{D(s)} \), where \( N(s) = N_0 + N_1s + ... \) and \( D(s) = D_0 + D_1s + ... \)
- The desired characteristic polynomial for the closed loop transfer function is \( D_0(s) = F_0 + F_1s + F_2s^2 + ... \)
- To determine the equations to solve set
  \[ A(s)D(s) + B(s)N(s) = D_0(s) \]
  and equate coefficients of \( s \). This will give you the system of equations to solve.

- To use the compensator to make a system a type one system, set \( A_0 = 0 \)

4) For the plant

\[
G_p(s) = \frac{1}{s(s+2)}
\]

show that the first order compensator that will put the closed loop poles at \(-1 \pm j\) and -3 is \( G_c(s) = 2 \).

5) For the plant

\[
G_p(s) = \frac{1}{s+2}
\]

Assume we want to place both closed loop poles at -4 and also have a type 1 system. Show that the first order compensator that will do this is given by

\[
G_c(s) = \frac{6(s+2.667)}{s}
\]

6) For a plant like the systems we have in lab with transfer function given by

\[
G_p(s) = \frac{6000}{s^2 + 3.2s + 400}
\]

a) Show that the first order compensator that places all three closed loop poles at -5 is given by

\[
G_c(s) = \frac{-0.0605s - 0.7658}{s + 11.8}
\]

Note that the numerator of this compensator make it pretty useless for tracking a step input.

b) Show that the second order system that places all four closed loop poles at -5 and produces a type 1 system is given by

\[
G_c(s) = \frac{-0.5063s^2 - 1.0367s + 0.1042}{s(s + 16.8)}
\]