If matrix $P$ is given as

$$P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Then

$$P^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

and the determinant of $P$ is given by $ad - bc$.

The general form for writing a continuous time state variable system is

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

Now assume we are using state variable feedback, so that $u(t) = f \bar{r}(t) - k^T \bar{x}(t)$. Here $\bar{r}(t)$ is our new reference input, $f$ is a scaling factor, and $k^T = [k_1 \ k_2]$ is the feedback gain matrix. With this state variable feedback, we have the system

$$\dot{\bar{x}}(t) = A\bar{x}(t) + B(f \bar{r}(t) - k^T \bar{x}(t))$$

or

$$\dot{\bar{x}}(t) = \tilde{A}\bar{x}(t) + \tilde{B}\bar{r}(t)$$

where $\bar{r}(t)$ is the new input. For $D = 0$, the transfer matrix is given by

$$G(s) = C\begin{bmatrix}(sI - \tilde{A})^{-1}\end{bmatrix}\tilde{B}$$

For each of the systems below,

- determine the controllability matrix and if the system is controllable
- determine the observability matrix and if the system is observable
- determine the transfer matrix
- determine the transfer matrix when there is state variable feedback
- determine if $k_1$ and $k_2$ exist to allow us to place the poles arbitrarily
1. Let
\[ A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [0 \ 1], D = 0 \]

Not Controllable, Observable, \( G(s) = \frac{(s-1)f}{(s-1)(s-1+k_2)} \)

2. Let
\[ A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [0 \ 1], D = 0 \]

Controllable, Not Observable, \( G(s) = \frac{sf}{s^2+(k_2-1)s+k_1} \)

3. Let
\[ A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \ 0], D = 0 \]

Controllable, Observable, \( G(s) = \frac{f}{s^2+(k_2-1)s+(k_1-1)} \)

4. Let
\[ A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = [0 \ 1], D = 0 \]

Not controllable, Observable, \( G(s) = \frac{(s+1)f}{(s+k_1)(s+k_2)-(k_1-1)(k_2-1)} \)