For a system with plant

\[ G_p(s) = \frac{s + 3}{s(s - 1)} \]

show that the quadratic optimal closed loop transfer function is

\[ G_0(s) = \frac{10(s + 3)}{s^2 + 12.7s + 30} \]

when \( q = 100 \).

What are \( e_p \) and \( e_v \) for this system? (Ans. \( e_p = 0 \), \( e_v = 0.09 \))

For a system with plant

\[ G_p(s) = \frac{s - 1}{s(s - 2)} \]

show that the quadratic optimal closed loop transfer function is

\[ G_0(s) = \frac{-10(s - 1)}{s^2 + 11.1s + 10} \]

when \( q = 100 \).

What are \( e_p \) and \( e_v \) for this system? (Ans. \( e_p = 0 \), \( e_v = 2.11 \))
For a one degree of freedom system like we have in lab, with plant

\[ G_p(s) = \frac{15}{0.0025s^2 + 0.0080s + 1} \]

a) Show that when \( q = 0.1 \) the quadratic optimal closed loop transfer function is

\[ G_0(s) = \frac{1856.6}{s^2 + 55.5s + 1939.1} \]

and the position error is \( e_p = 0.043 \).

b) Show that the controller is given by

\[ G_c(s) = \frac{0.0038s^2 + 0.012s + 1.5}{0.012s^2 + 0.67s + 1} \]

c) Using the *quadratic.m* program, plot the step response of this system for \( q = 0.01, q = 0.1, \) and \( q = 1 \). To use this program (which you’ll be using in lab this week), you first need to enter the estimated plant transfer function in the form

\[ G_p(s) = \frac{K_{clg} \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]

*quadratic.m* has the input arguments

- The amplitude of the step input (assume 1 cm, so enter 1)
- The plant transfer function \( G_p(s) \)
- The value of \( q \).
- The length of time to plot the results (be sure the system has reached steady state, but not too long).
- The filename with data to compare the model to. In this case, type ” (two single quotes). In lab you’ll generate data for this part.

You should see that as \( q \) increases, which means the penalty on the difference between input and output is getting larger, the system should produce a smaller and smaller position error and response more and more quickly. If your final position error is not near 0, you’ve probably made a scaling mistake.
For the systems on the following page:

a) Determine the system type.

b) If the system is type 0 assume $G_{pf} = 1$ and determine the position error constant $K_p$ and the position error $e_p$. Then determine the value of $G_{pf}$ that makes the position error zero.

c) If the system is type 1, assume $G_{pf} = 1$ and determine the position error, the velocity error constant $K_v$, and the velocity error $e_v$. Is there any constant value of $G_{pf}$ that can change the velocity error?

Ans. $e_p = \frac{3}{5}$ and $G_{pf} = \frac{5}{2}$, $e_p = \frac{3}{13}$ and $G_{pf} = \frac{13}{10}$, $e_v = \frac{3}{5}$, $e_v = \frac{4}{5}$, $G_{pf}$ has no effect on $e_v$. 