Temperature

Fundamental meaning of temperature

Temperature is a measure of the average kinetic energy possessed by a group of particles.

For example, if the temperature of a group of particles is $T$ (absolute temperature, in °K), the average energy, per degree of freedom is

$$E_{av} = \frac{1}{2} kT,$$

where $k$ is Boltzmann’s constant.

Practical measurement

Practical temperature measurement usually involves measuring some physical quantity that varies in a repeatable fashion with temperature. Temperature transducers use quantities like volume, resistance, and voltage to produce quantities that can be used as analogs to the actual temperature.

<table>
<thead>
<tr>
<th></th>
<th>thermocouple</th>
<th>RTD resistance temperature detector</th>
<th>thermister</th>
<th>I.C. sensor</th>
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</thead>
<tbody>
<tr>
<td>advantages</td>
<td>&gt;wide range</td>
<td>&gt;most stable</td>
<td>&gt;sensitive</td>
<td>&gt;most linear</td>
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<td>&gt;inexpensive</td>
<td>&gt;most accurate</td>
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<td>&gt;most sensitive</td>
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<td>&gt;self-powered</td>
<td>&gt;more linear than TC</td>
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<td>&gt;inexpensive</td>
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<td></td>
<td>&gt;rugged</td>
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<td></td>
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<tr>
<td>disadvantages</td>
<td>&gt;least stable</td>
<td>&gt;expensive</td>
<td>&gt;nonlinear</td>
<td>&gt;very limited range</td>
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<td>&gt;nonlinear</td>
<td>&gt;needs external power</td>
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<td>&gt;reference required</td>
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Thermocouples output a voltage analog; an RTD produces a resistance analog; the thermistor gives a resistance analog, and I.C. sensors are available that output a voltage or current.

We are going to take a relatively close look at the RTD and the thermocouple. The RTD is known for its stability and accuracy, the thermocouple for its wide range and its ruggedness. The thermocouple is the workhorse of the lot and a sensor that is widely misused.
Resistance temperature detectors (RTDs)

The resistance of metals increases with temperature. In an RTD, the resistance of resistor, typically a platinum $100\,\Omega$ (at $0^\circ\text{C}$) resistor, is used as an analog to temperature. Platinum is the most popular metal due to its linearity and its stability.

In 1983, the International Electrotechnical Commission (IEC) adopted the Deutsche Institute of Normung (DIN) standard platinum RTD with a temperature coefficient of $\alpha = 0.00385\,\Omega/\Omega/{}^\circ\text{C}$. The U.S. Industrial Standard is $\alpha = 0.003902\,\Omega/\Omega/{}^\circ\text{C}$.

For $T$ near $0^\circ\text{C}$

$$R \equiv 100\,\Omega \left( 1 + \alpha T \right) \quad \text{where } T \text{ is in } {}^\circ\text{Celsius}$$

Otherwise

A relation that remains close at higher temperatures is the Callandar-Van Dusen equation

$$R_{\text{RTD}} = R_0 \left[ 1 + \alpha \left[ T - \delta (0.01T-1)(0.01T) - \beta (0.01T-1)(0.01T)^3 \right] \right]$$

In 1968, a more accurate equation was developed ($20^{\text{th}}$ order). Tables are often used (you will in the homework). The tables in your textbook are for the U.S. Industrial Standard. In the lab, we will use the more popular DIN standard RTDs.

The platinum RTD is used as the primary element in all high-accuracy resistance thermometers. It is used as the interpolation standard from the boiling point of oxygen (-182.96°C) to the melting point of antimony (630.74°C).
Signal Conditioning Requirements for the RTD

The RTD is often used in a one-active-arm Wheatstone bridge, which serves to transform the changes in resistance to changes in voltage.

In this arrangement, the resistance of the leads can cause inaccuracies in the temperature measurement. We can eliminate this source of error by using three leads to connect the RTD to the bridge.

Three lead arrangement

In this arrangement, $R_{\text{lead}}$ appears in $R_3$ and in $R_4$ so does not affect the voltage divider.
Prove this for yourself. Find $V_b$.

The lead resistance in series with the voltmeter does not affect $V_b$ too much since the resistance of a voltmeter is high. This high resistance will cause the current through the voltmeter to be low and therefore the voltage drop across $R_{\text{lead}}$ will be low.

This arrangement works very well as long as the temperatures being measured do not vary much. If they do then, $R_g$ will not always be close to the value of $R$ and the inherent nonlinear nature of the Wheatstone bridge will become a problem.

Example
Assume $R_{\text{lead}} = 2\Omega$.

i) Estimate the measurement error when using 3-wire 100Ω RTD ($\alpha = 0.00385 \Omega/\Omega/^{\circ}\text{C}$) when $V_b/V_s = 1.942(10^{-3})$. That is, estimate the difference in temperature extracted when $R_{\text{lead}}$ is neglected and when $R_{\text{lead}}$ is accounted for.

ii) Compare this error the error present when measuring 100°C.
4-Wire Measurement
To eliminate the problem of the inherent nonlinearity of the Wheatstone bridge, a constant current source can be used to transform the RTD’s change in resistance to a change in voltage and use a 4-wire voltage measurement to focus almost solely on the resistance of the RTD.

Using a constant current source \( (I_s) \), the change in \( R_g \) is transformed to \( V_g \). Note: Since there hardly any current flowing in the, it measures a voltage very close to \( V_g \).

The 4-wire measurement is a standard precision measurement technique. Its advantage is accuracy; its weakness is cost since it requires a constant current source.

Constant current source
There are many circuits that approximate a constant current source. Each realization has constraints or limits to the conditions under which it approximates a constant current source. Here is one using an op-amp.