INTRODUCTION TO LOAD-FLOW

Load-flow studies are probably the most common of all power system analysis calculations. They are used in planning studies to determine if and when specific elements will become overloaded. Major investment decisions begin with reinforcement strategies based on load-flow analysis. In operating studies, load-flow analysis is used to ensure that each generator runs at the optimum operating point; demand will be met without overloading facilities; and maintenance plans can proceed without undermining the security of the system.

The objective of any load-flow program is to produce the following information:

- Voltage magnitude and phase angle at each bus.
- Real and reactive power flowing in each element.
- Reactive power loading on each generator.

The above objectives are achieved by supplying the load-flow program with the following information:

- Branch list of the system connections i.e., the impedance of each element, sending-end and receiving-end node #. Lines and transformers are represented by their $\pi$-equivalent models.
- Voltage magnitude and phase-angle at one bus, which is the reference point for the rest of the system.
- Real power generated and voltage magnitude at each generator bus.
- Real and reactive power demanded at each load bus.

The foregoing information is generally available since it either involves readily known data (impedances etc.) or quantities which are under the control of power system personnel (active power output and excitation of generators.)

Simply stated the load-flow problem is as follows:

- At any bus there are four quantities of interest: $\left| V \right|$, $\theta$, $P$, and $Q$.
- If any two of these quantities are specified, the other two must not be specified otherwise we end up with more unknowns than equations.
- Because records enable the real and reactive power to be accurately estimated at loads, $P$ and $Q$ are specified quantities at loads, which are called $PQ$ buses.
- Likewise, the real power output of a generator is controlled by the prime mover and the magnitude of the voltage is controlled by the exciter, so and $P$ and $\left| V \right|$ are specified at generators, which are called $PV$ buses.
- This means that $\left| V \right|$ and $\theta$ are unknown at each load bus and $\theta$ and $Q$ are unknown at each generator bus.
- Since the system losses are unknown until a solution to the load-flow problem has been found, it is necessary to specify one bus that will supply these losses. This is called the slack (or swing, or reference) bus and since $P$ and $Q$ are unknown, $\left| V \right|$ and $\theta$ must be specified. Usually, an angle of $\theta = 0$ is used at the slack bus and all other bus angles are expressed with respect to slack.
The foregoing is summarized in the following one-line diagram in which the specified quantities are italicized, while the quantities that are free to vary during the iteration process are indicated with up-and-down arrows. Note that at each bus we can write TWO node equations.

There are two major iterative techniques:

A. **The Gauss-Seidel Method**
   The Gauss-Seidel method is based on substituting nodal equations into each other. It is the slower of the two but is the more stable technique. It's convergence is said to be Monotonic. The iteration process can be visualized for two equations:

   Although not the best load-flow method, Gauss-Seidel is the easiest to understand and was the most widely used technique until the early 1970s.
Some software packages use Gauss-Seidel to start the solution process and then switch over to:

B. The Newton-Raphson Method

The Newton-Raphson method is the most efficient load-flow algorithm. The basic (no approximations) Newton-Raphson algorithm is based on the formal application of a well-known algorithm for the solution of a set of simultaneous non-linear equations of the form:

\[ [F(x)] = [0] \]

Where: \([F(x)]\) is a vector of functions: \(f_1 \ldots f_n\) in the variables \(x_1 \ldots x_n\).

The above expression does not equal zero until the Newton-Raphson process has converged (i.e., all the \(x\)'s have been found) and the iterations have to be performed, starting at some initial set of values \(x_1, x_2, \ldots x_n\). In the load-flow problem the \(x\)'s are voltage magnitude and phase angle at all PQ buses and voltage phase angles at all PV buses i.e., angles at all buses except slack and \(|V|\) for all load buses.

The iterations are performed by linearizing the non-linear equations \(F(x) = [0]\) and adjusting the values of \(x\). This process can be visualized in the case of a single-variable problem, which could be formed by subtracting the two equations used at the beginning of the Gauss-Seidel section, i.e.

\[ f(x) = \text{eqn 1} - \text{eqn 2} \quad \{f(x) = 0 \text{ at the solution}\} \]

\[ f(x) = 0 \]

is the required solution.

The initial estimate is \(x^0 \approx x + \Delta x\). This can be improved by applying trigonometry once the function has been differentiated.

An estimate for \(\Delta x\) is obtained from:

\[ \Delta x^0 = \frac{f(x + \Delta x)}{df(x + \Delta x)/dx} \]

Then the estimate of \(x\) is improved by:

\[ x^1 = x^0 - \Delta x^0 \]
Information available from load-flow studies

The basic information contained in the load-flow output is:

i) All bus voltage magnitudes and phase angles w.r.t the slack bus.
ii) All bus active and reactive power injections.
iii) All line sending- and receiving-end complex power flows.
iv) Individual line losses can be deduced by subtracting receiving-end complex power from sending-end complex power.
v) Total system losses can be deduced by summing item iv) for all lines, or by summing complex power at all loads and generators and subtracting the totals.

The most important information obtained from the load-flow is the voltage profile of the system. If $|V|$ varies greatly over the system, large reactive flows will result; this, in turn, will lead to increased real power losses and, in extreme cases, an increased likelihood of voltage collapse. When a particular bus has an unacceptably low voltage, the usual practice is to install capacitor banks in order to provide reactive compensation to the load. Load-flow studies are used to determine how much reactive compensation should be applied at a PQ bus, to bring its voltage up to an appropriate level, i.e.:

i) Re-execute the load-flow with the bus re-designated as PV type with the required voltage level specified.
ii) Subtract the value of $Q$ obtained from i) from the value obtained in the old load-flow when the bus was PQ.
iii) The result is the value of $Q_c$ needed to bring the voltage up to the specified level. Note that if the specified voltage is not 1 pu, then the value of $Q_c$ has to be adjusted by $1/|V|^2$ in order to specify $Q_c$ at rated voltage.

If new lines (or additional transformers) are to be installed, to reinforce the system, a load-flow will show how it will relieve overloads on adjacent lines. It will also show how much reduction in losses will result from the new line (important for economic assessment.)