Reflected Waves

\[ \Gamma = \left. \frac{V'}{V} \right|_{z=0} \quad T = \left. \frac{V'}{V} \right|_{z=0} \]

The “boundary conditions” are KVL and KCL

\[ \Gamma = \frac{Z_{c2} - Z_{c1}}{Z_{c2} + Z_{c1}} \quad T = \frac{2Z_{c2}}{Z_{c2} + Z_{c1}} \]
Lumped-element loads
The reflection coefficient would be unchanged if, instead of a second medium with characteristic impedance $Z_{c2}$, there were a lumped element with this same impedance, $Z_L = Z_{c2}$.

Matched load ($Z_L = Z_{c1}$)
$\Gamma = 0$, no reflections.

Short circuit ($Z_L = 0$)
$\Gamma = -1$, reflected voltage cancels incident voltage ($V_1 = 0$)

Open circuit ($Z_L = \infty$)
$\Gamma = 1$, total voltage is twice incident voltage
(Or, why you don’t want to be at the end of the power line in an area with frequent lightning storms)

$\Gamma$ for a passive load is a complex number with a magnitude less than or equal to one.
Transients waves on lossy lines

When loss must be taken into account, high frequencies see a different environment due to the frequency dependence of the loss mechanism (skin effect and dielectric loss). This complicates the analysis for the transient solution.

The text treats the important special case of skin effect being the dominant loss mechanism with a step function input.

The result is that the response to a step function input is

\[ v(z,t) = \text{erfc} \left( \frac{Z \sqrt{\mu z}}{2P \sqrt{t - \frac{z}{v_p}}} \right) \]

where \( \text{erfc} \) is the complementary error function.

The most important things here are its qualitative characteristics and the physical reasons for these characteristics.

Dispersion and attenuation

In general, \( \alpha \) and \( v_p \) are functions of \( \omega \) in a lossy transmission line.

It is true that, in the low-loss approximation, \( v_p \) is not a function of \( \omega \), but, strictly speaking, \( v_p \) is a function of \( \omega \), even in the low-loss case (it's a second-order effect in the low-loss case).

Dispersion occurs when \( v_p \) is a function of \( \omega \). If one looks sufficiently close, \( v_p \) is almost always dependent on \( \omega \). Aside from effects due to loss, \( \epsilon \) and \( \lambda \) vary with frequency, or, more fundamentally, \( \epsilon \) and \( \mu \) vary with frequency.

For transmission lines, \( \alpha \) typically increases with \( \omega \) (\( R_{ac} \) increases as \( \omega \) increases).

From Fourier analysis, the harmonic functions form a complete basis set, and we can use superposition to construct any waveform. That is, we can expand any signal in terms of harmonic functions.
Looking at the problem in this light, there are two things going one simultaneously: 1st, the time-delay suffered by components varies with their frequency, and 2nd, the high-frequency components of waveforms suffer greater attenuation as they propagate.

The net result is that waves (any wave that is not a pure sinusoid) changes shape as it propagates. Sharp corners become rounded, rise times become longer, fall times lengthen.